

Computer Graphics

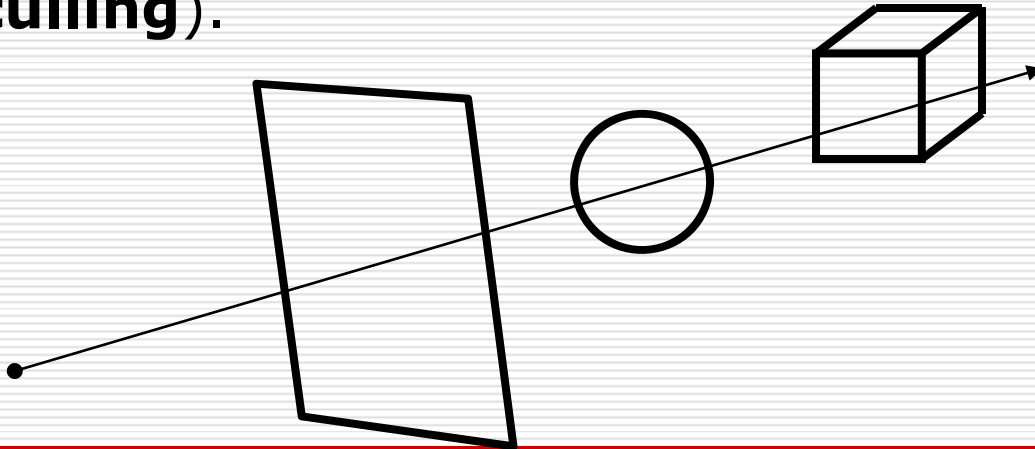
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National Taiwan University

Visible-Surface Determination

- ❑ Back-Face Culling
 - ❑ The Depth-Sort Algorithm
 - ❑ Binary Space-Partitioning Trees
 - ❑ The z-Buffer Algorithm
 - ❑ Scan-Line Algorithm
 - ❑ Visible-Surface Ray Tracing
(Ray Casting)
 - ❑ Warnock's Algorithm
-

Visible-Surface Determination = Hidden Surface Removal

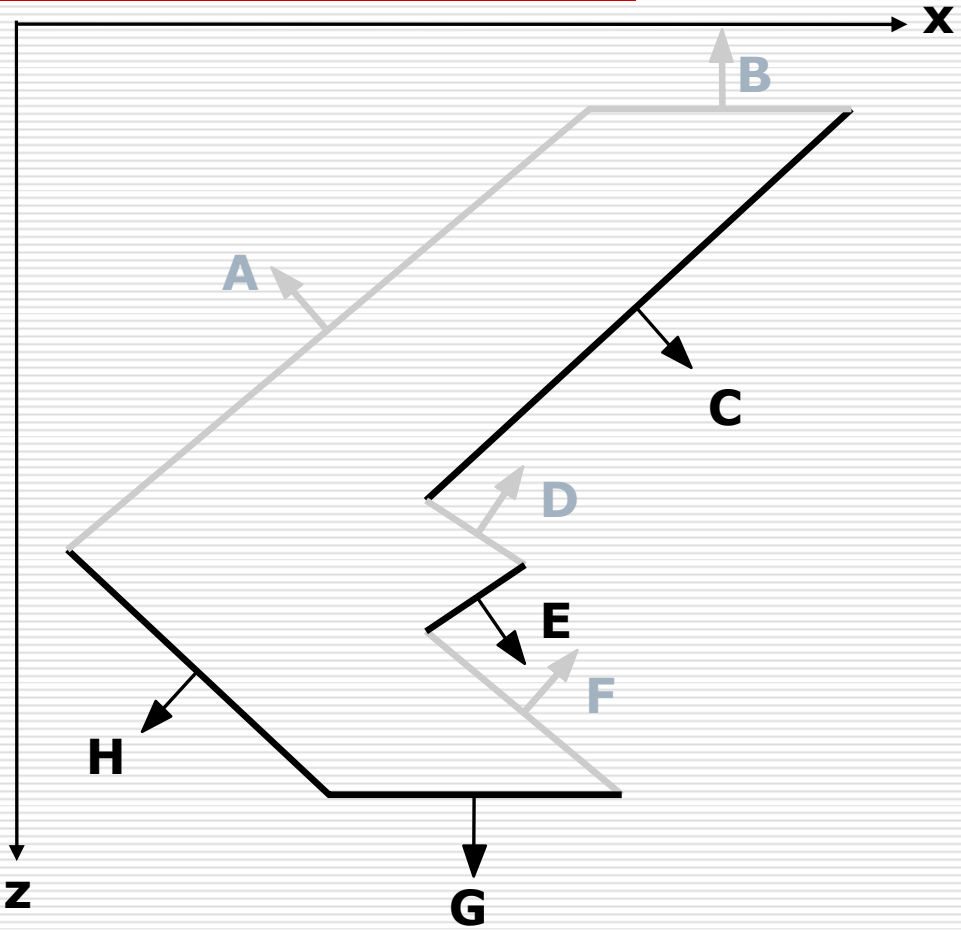
- ❑ Determining what to render at each pixel.
- ❑ A point is visible if there exists a direct line-of-sight to it, unobstructed by another other objects (**visible surface determination**).
- ❑ Moreover, some objects may be invisible because there are behind the camera, outside of the field-of-view, too far away (**clipping**) or back faced (**back-face culling**).



Hidden Surfaces: why care?

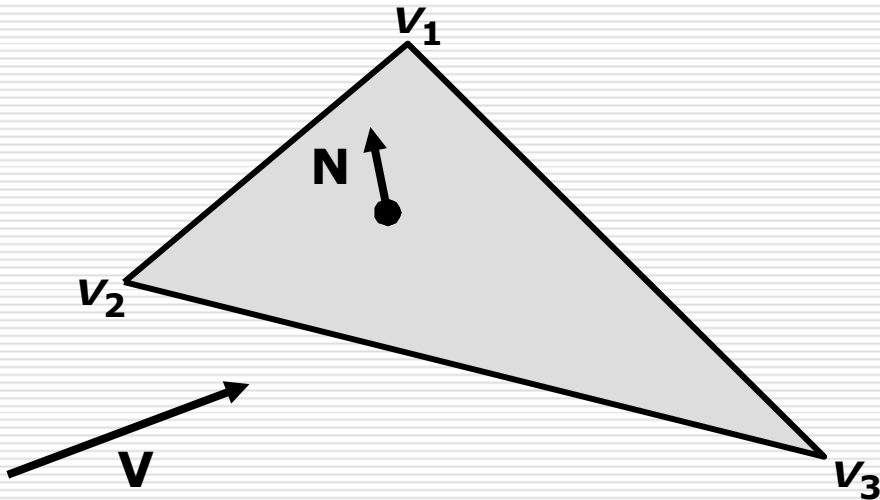
- **Occlusion:** Closer (opaque) objects along same viewing ray obscure more distant ones.
 - Reasons for removal
 - Efficiency: As with clipping, avoid wasting work on invisible objects.
 - Correctness: The image will look wrong if we don't model occlusion properly.
-

Back-Face Culling = Front Facing



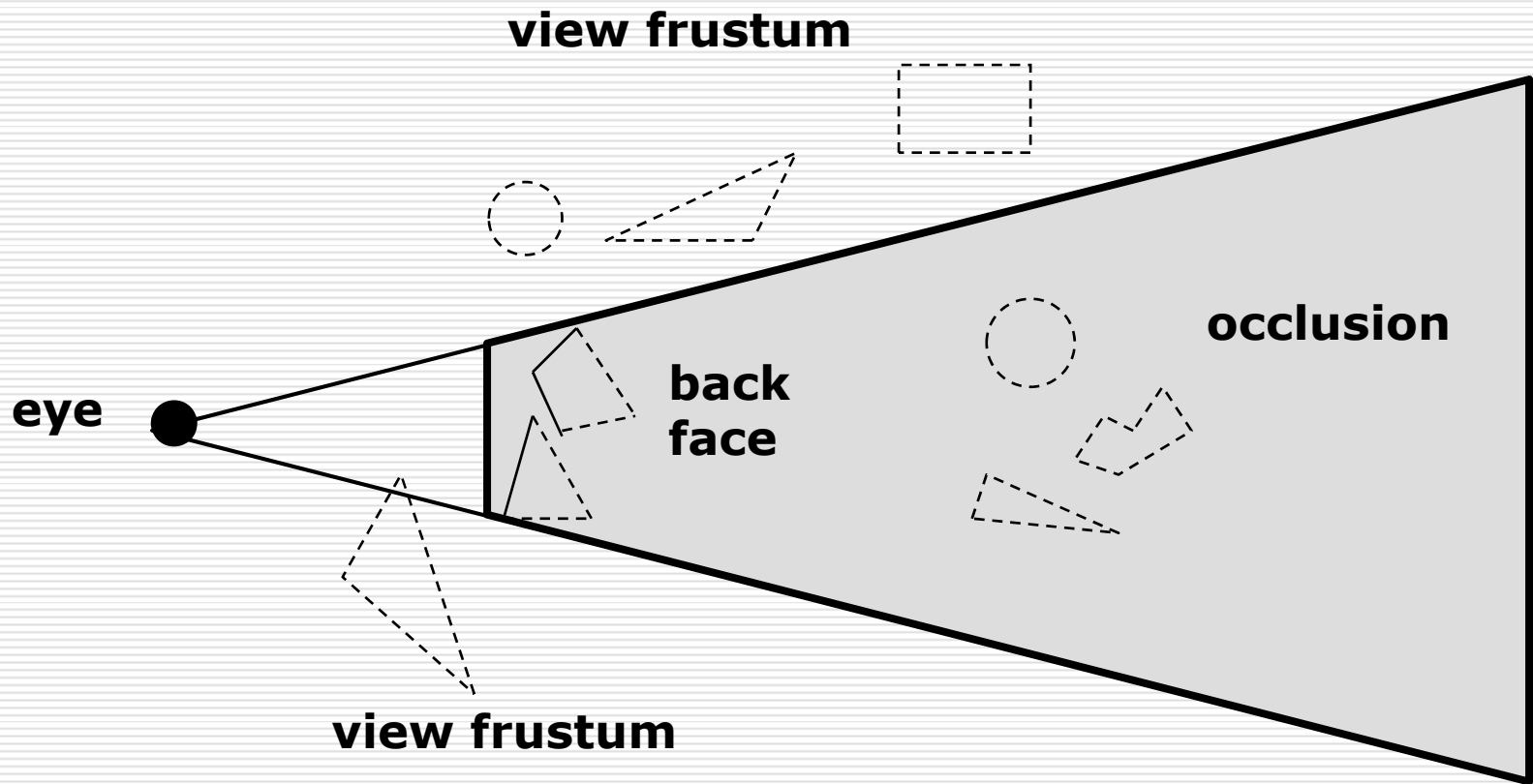
Back-Face Culling = Front Facing

- use cross-product to get the normal of the face (not the actual normal)
- use inner-product to check the facing



$$N = (v_2 - v_1) \times (v_3 - v_1)$$

Clipping (View Frustum Culling)

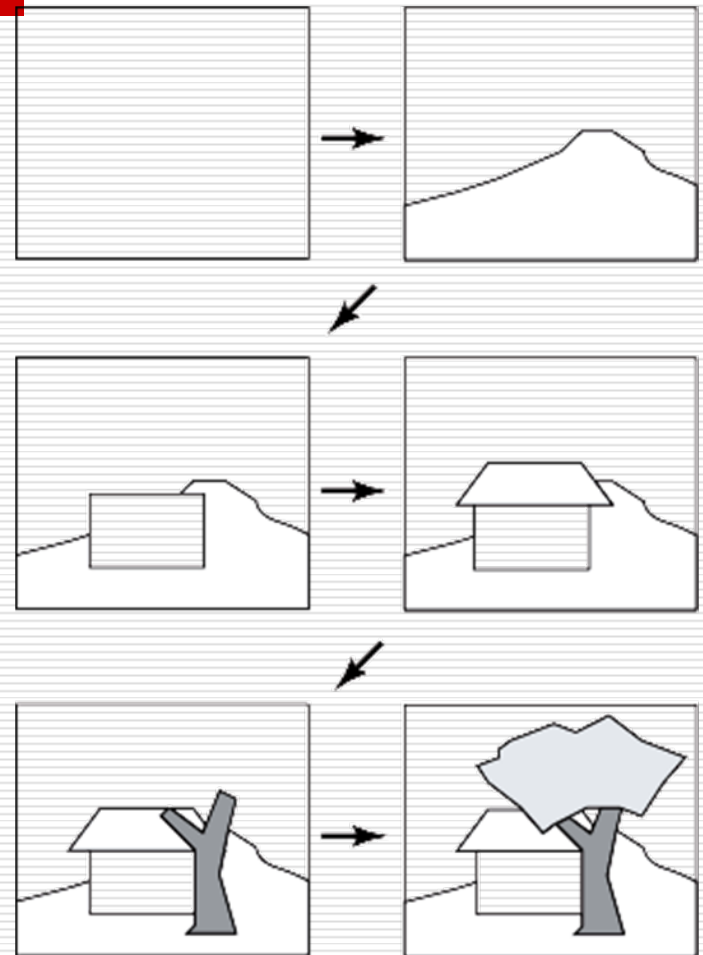


List-Priority Algorithms

- The Painter's Algorithm
 - The Depth-Sort Algorithm
 - Binary Space-Partitioning Trees
-

The Painter's Algorithm

- Draw primitives from back to front need for depth comparisons.



The Painter's Algorithm

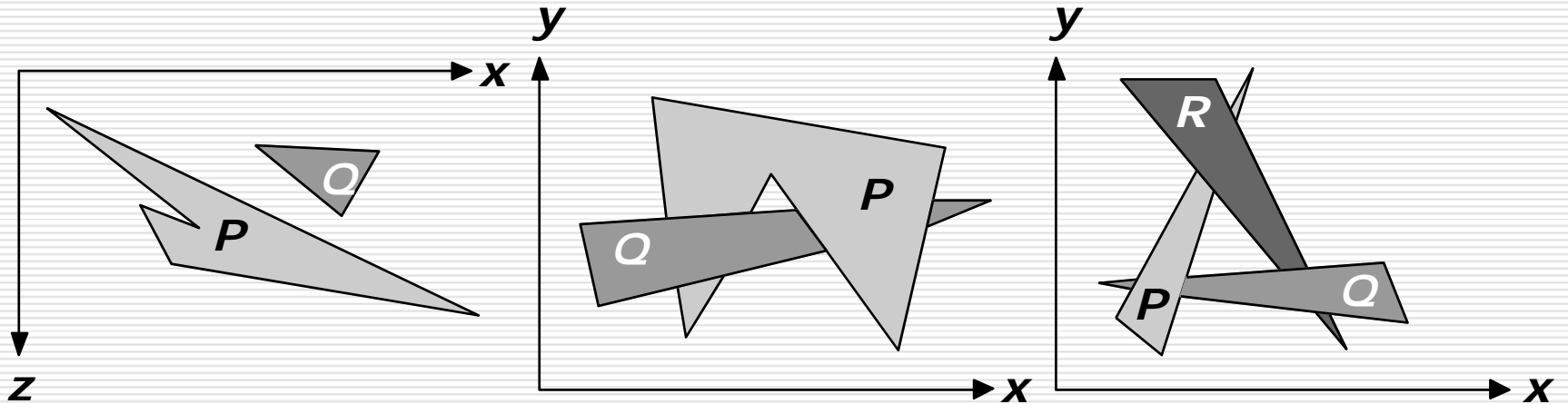
- for the planes with constant z
 - not for real 3D, just for 2½D

 - **sort** all polygons according to the smallest (farthest) z coordinate of each
 - **scan convert** each polygon in ascending order of smallest z coordinate (i.e., back to front)
-

The Depth-Sort Algorithm

- **sort** all polygons according to the smallest (farthest) z coordinate of each
 - resolve any ambiguities that sorting may cause when the polygons' z extents **overlap, splitting** polygons if necessary
 - **scan convert** each polygon in ascending order of smallest z coordinate (i.e., back to front)
-

Overlap Cases

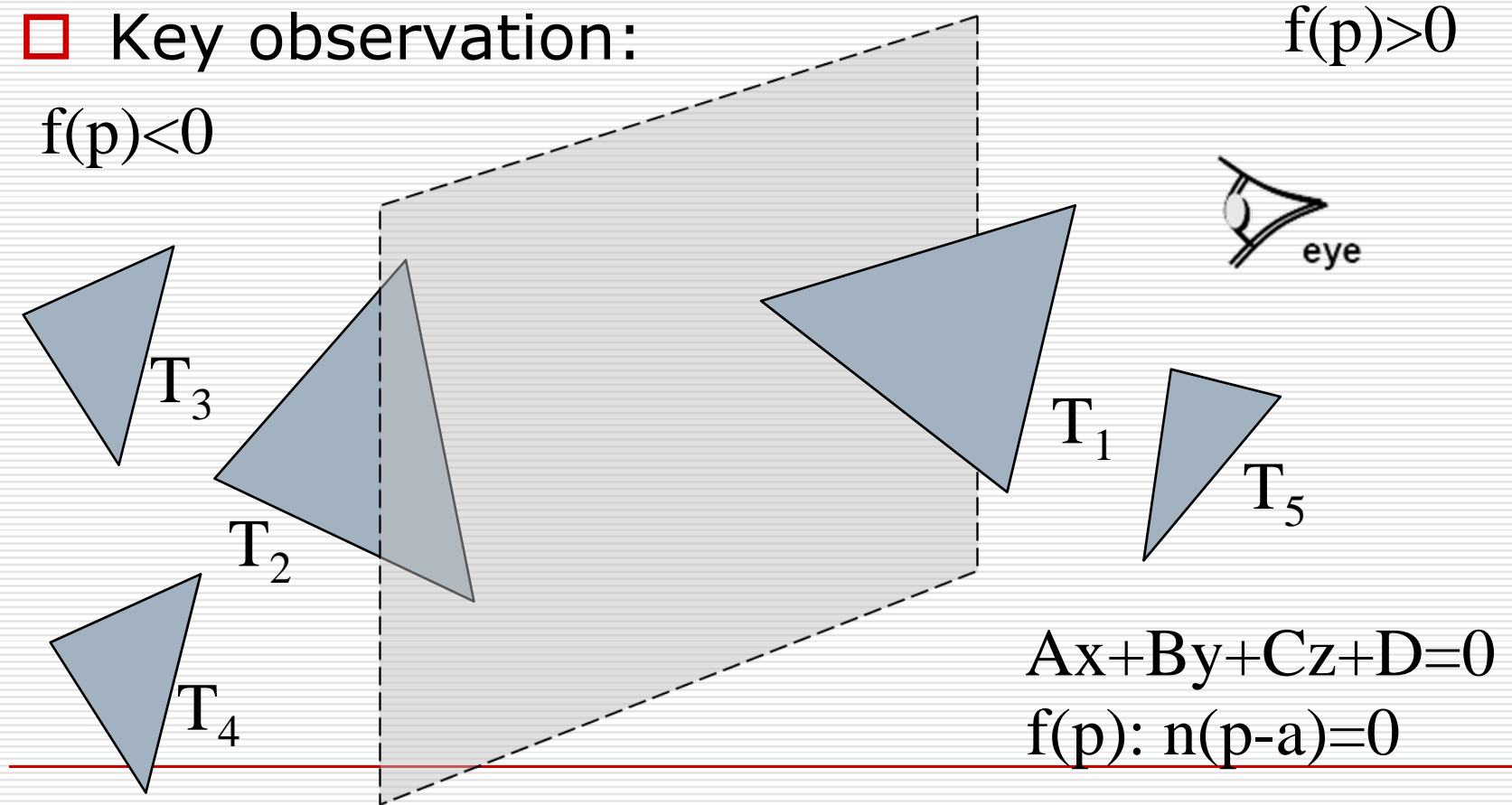


Overlap Detection

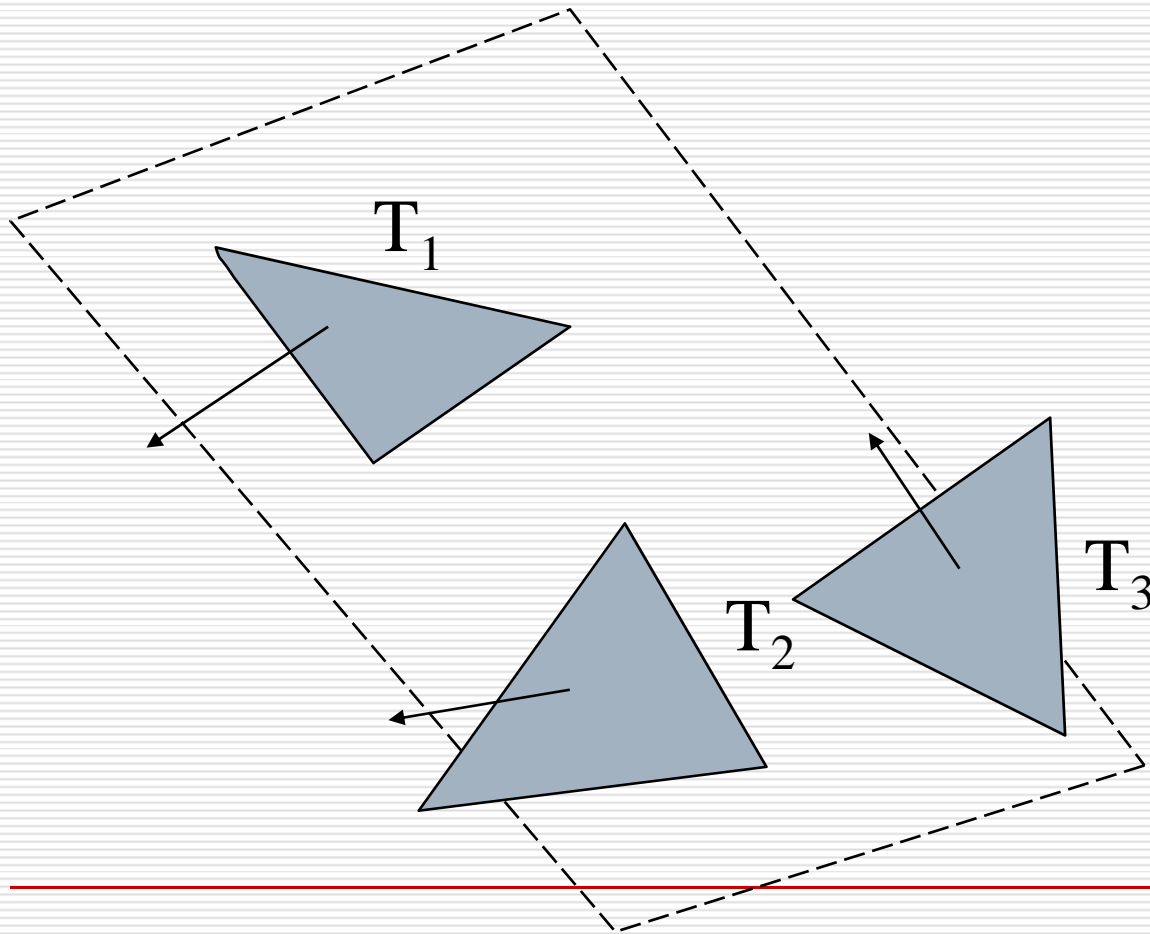
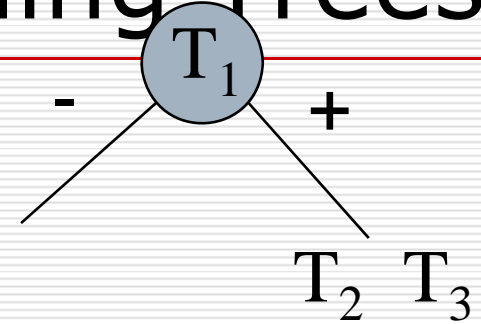
- Do the polygons'x not overlap?
 - Do the polygons'y not overlap?
 - Is P entirely on the opposite side of Q's plane from the viewpoint?
 - Is Q entirely on the same side of P's plane as the viewpoint?
 - Do the projections of the polygons onto the (x,y) plane not overlap?
-

Binary Space-Partitioning Trees

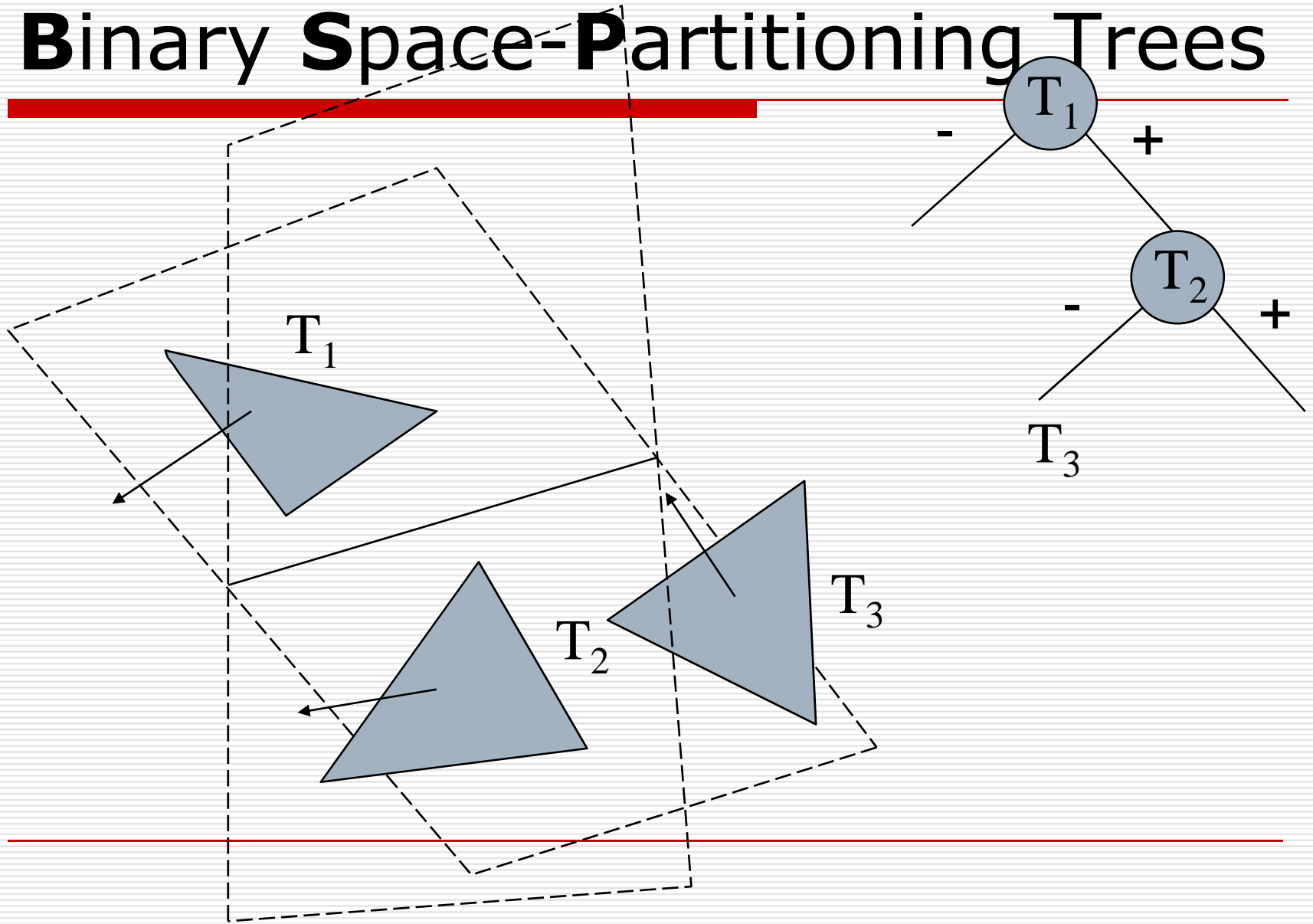
- An improved painter's algorithm
- Key observation:



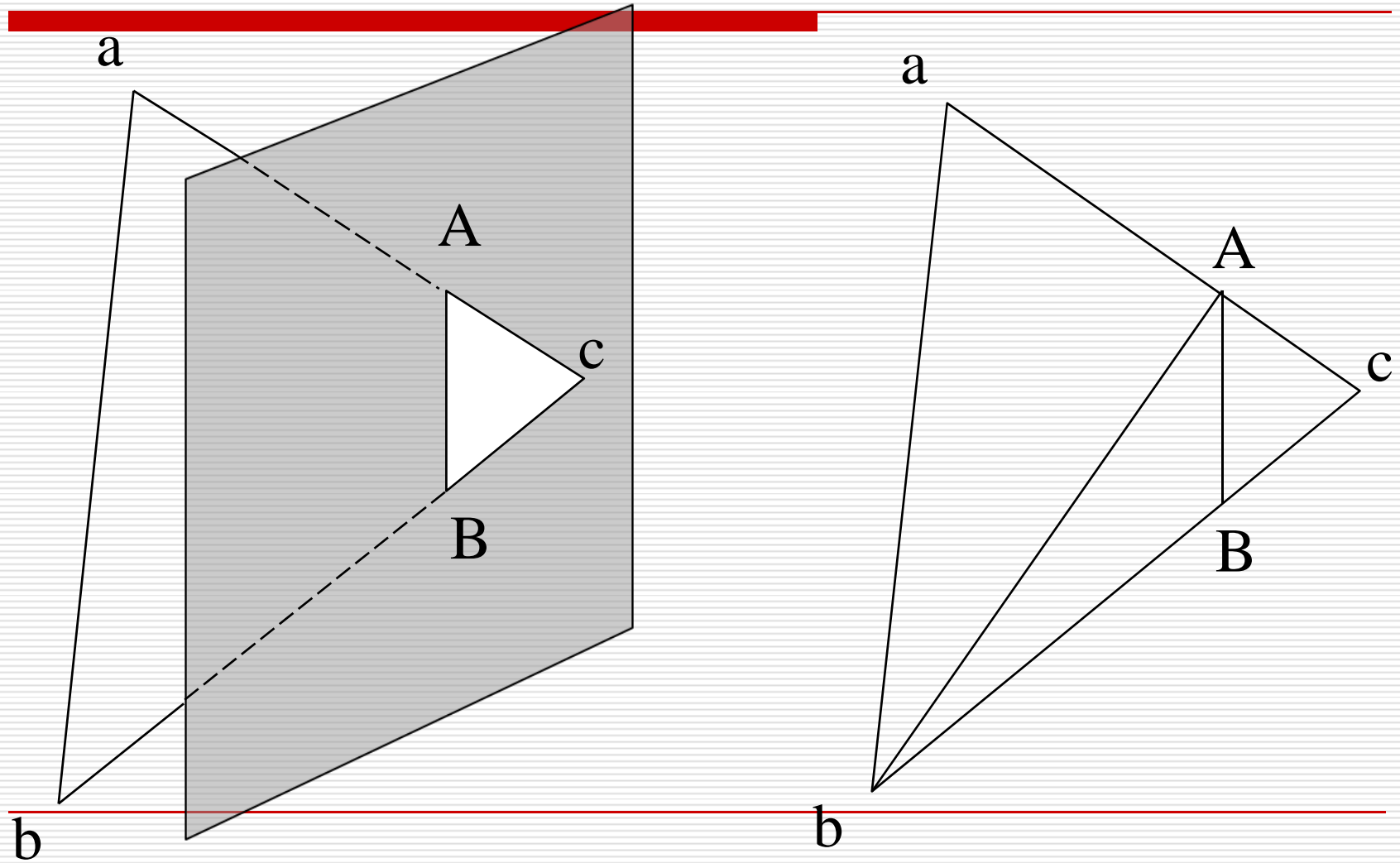
Binary Space-Partitioning Trees



Binary Space-Partitioning Trees



Splitting triangles



BSP Tree Construction

```
BSPtree makeBSP(L: list of polygons) {  
    if (L is empty) {  
        return the empty tree;  
    }  
    Choose a polygon P from L to serve as root;  
    Split all polygons in L according to P  
    return new TreeNode (  
        P,  
        makeBSP(polygons on negative side of P),  
        makeBSP(polygons on positive side of P))  
}
```

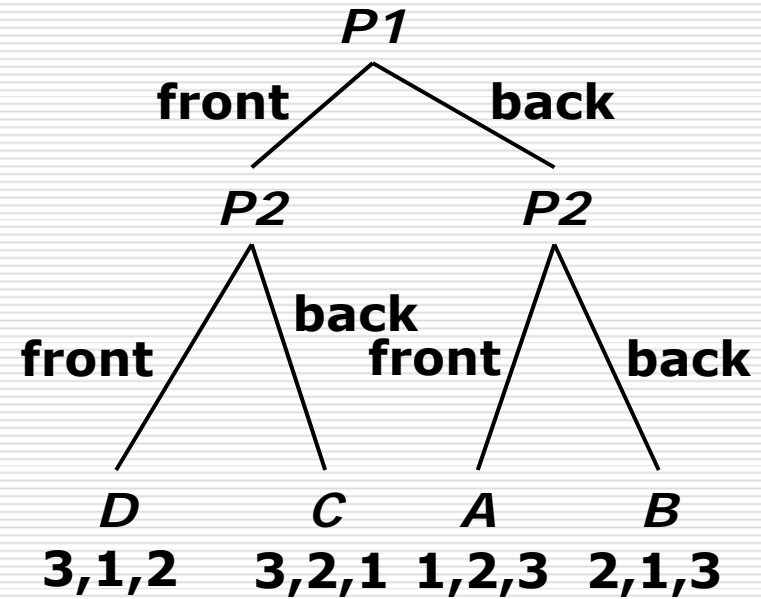
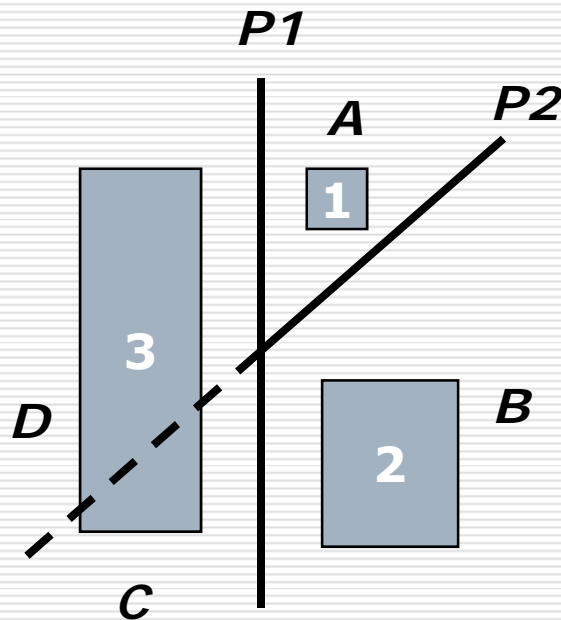
- Splitting polygons is expensive! It helps to choose P wisely at each step.
 - Example: choose five candidates, keep the one that splits the fewest polygons.
-

BSP Tree Display

```
void showBSP(v: Viewer, T: BSPtree) {  
    if (T is empty) return;  
    P = root of T;  
    if (viewer is in front of P) {  
        showBSP(back subtree of T);  
        draw P;  
        showBSP(front subtree of T);  
    } else {  
        showBSP(front subtree of T);  
        draw P;  
        showBSP(back subtree of T);  
    }  
}
```

[2D BSP demo](#)

Binary Space-Partitioning Trees

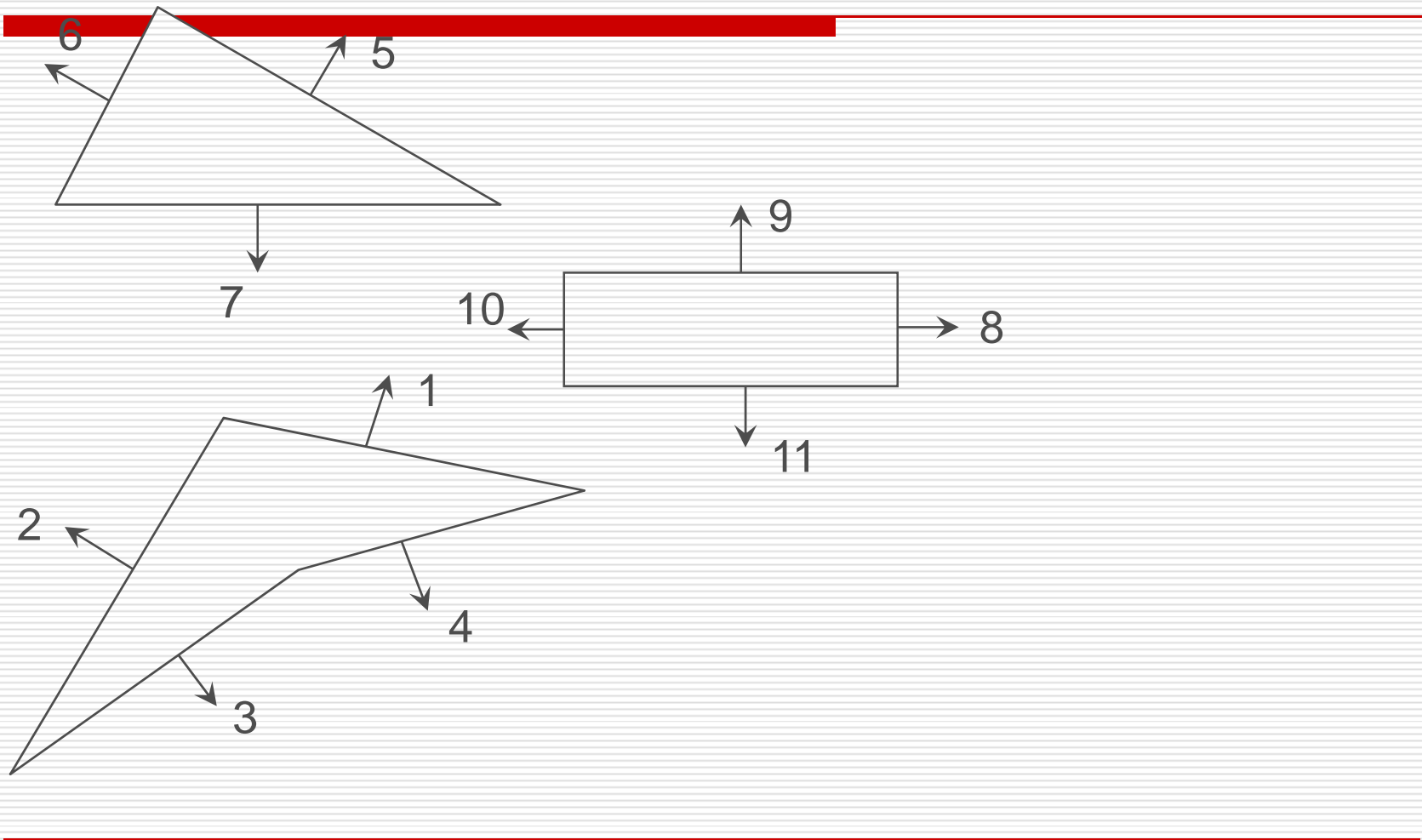


- extremely efficient for static objects
-

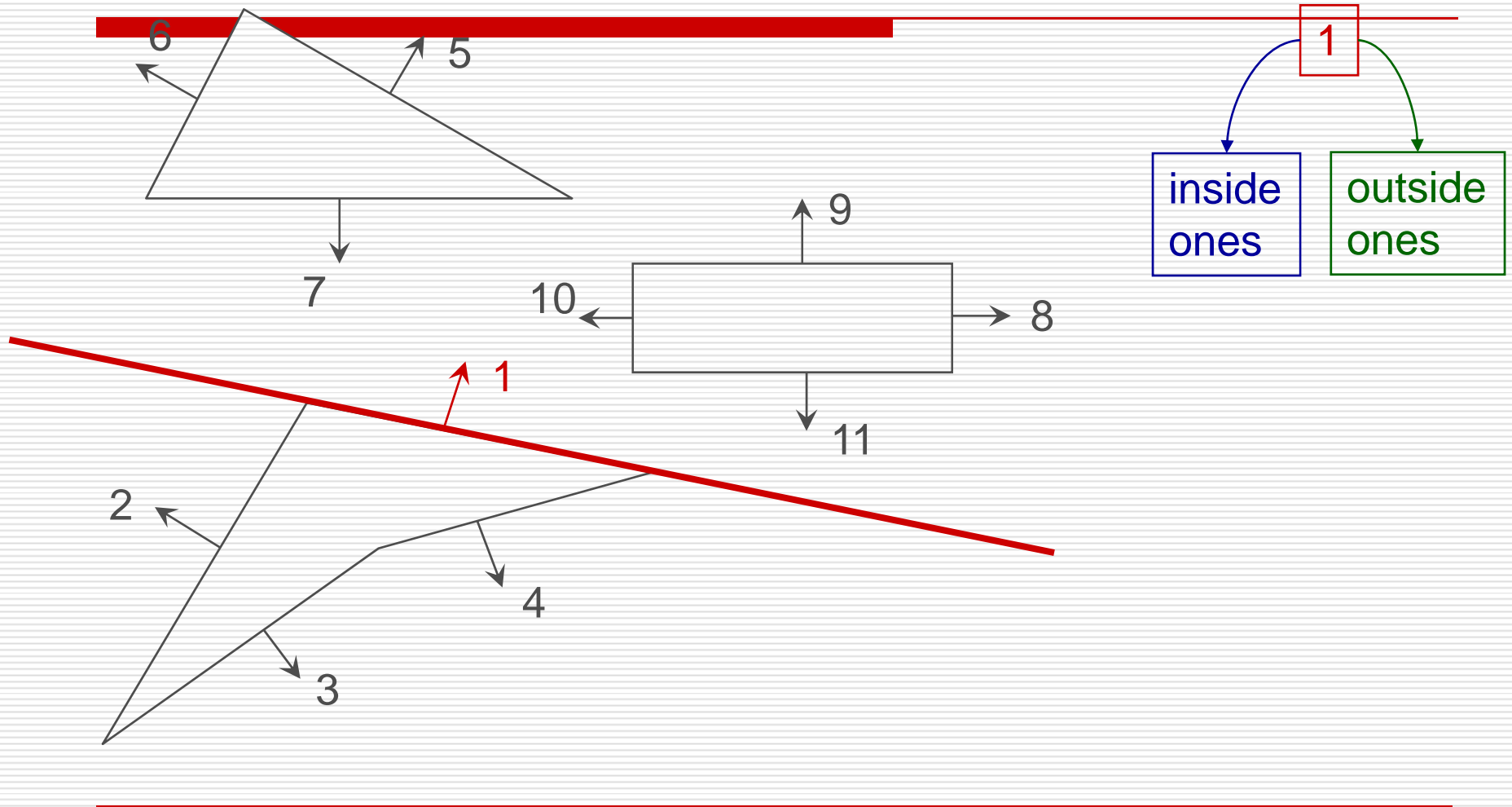
Binary Space-Partitioning Trees

- ❑ Same BSP tree can be used for any eye position, constructed only once if the scene is static.
 - ❑ It does not matter whether the tree is balanced. However, splitting triangles is expensive and try to avoid it by picking up different partition planes.
-

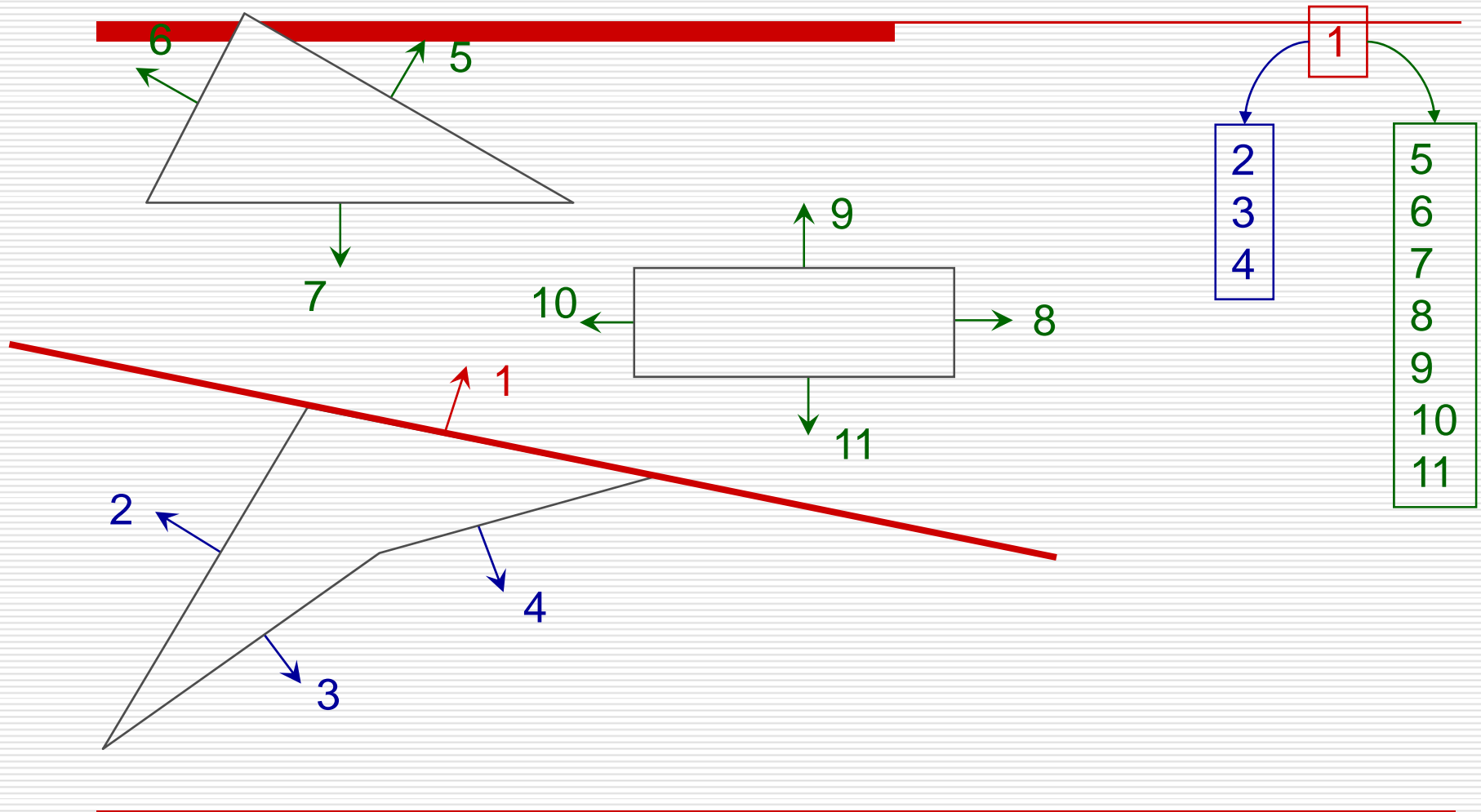
BSP Tree



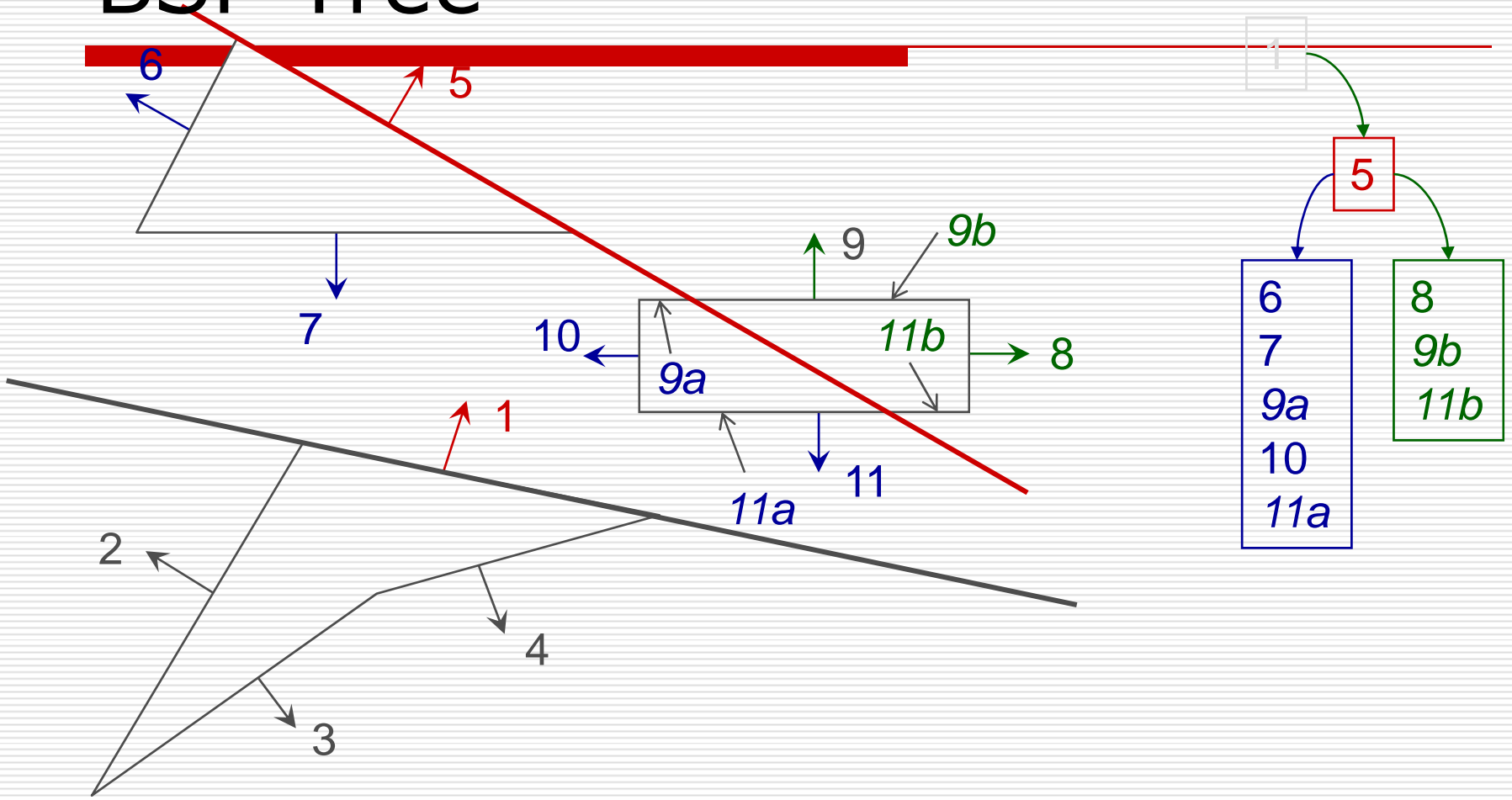
BSP Tree



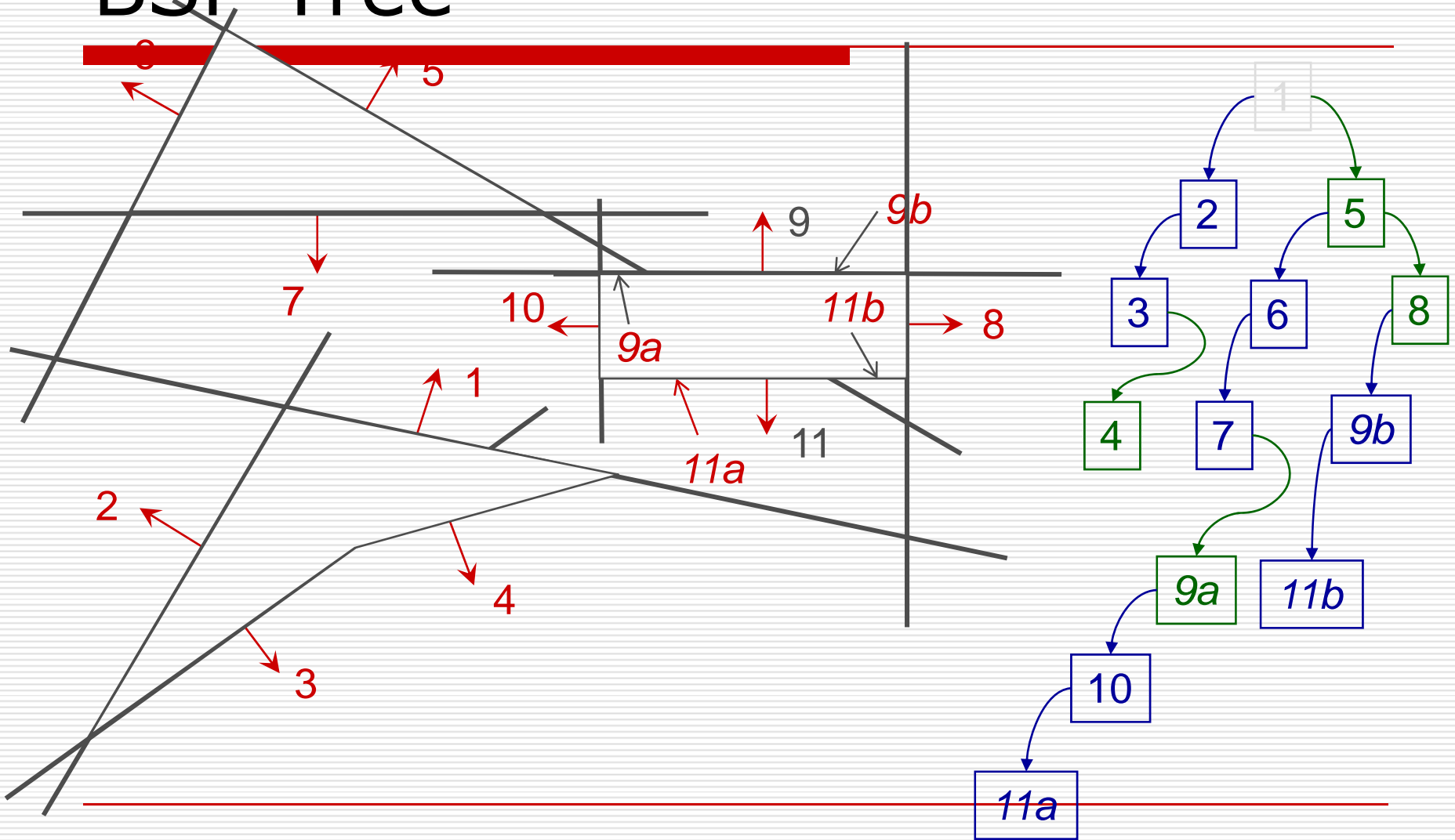
BSP Tree



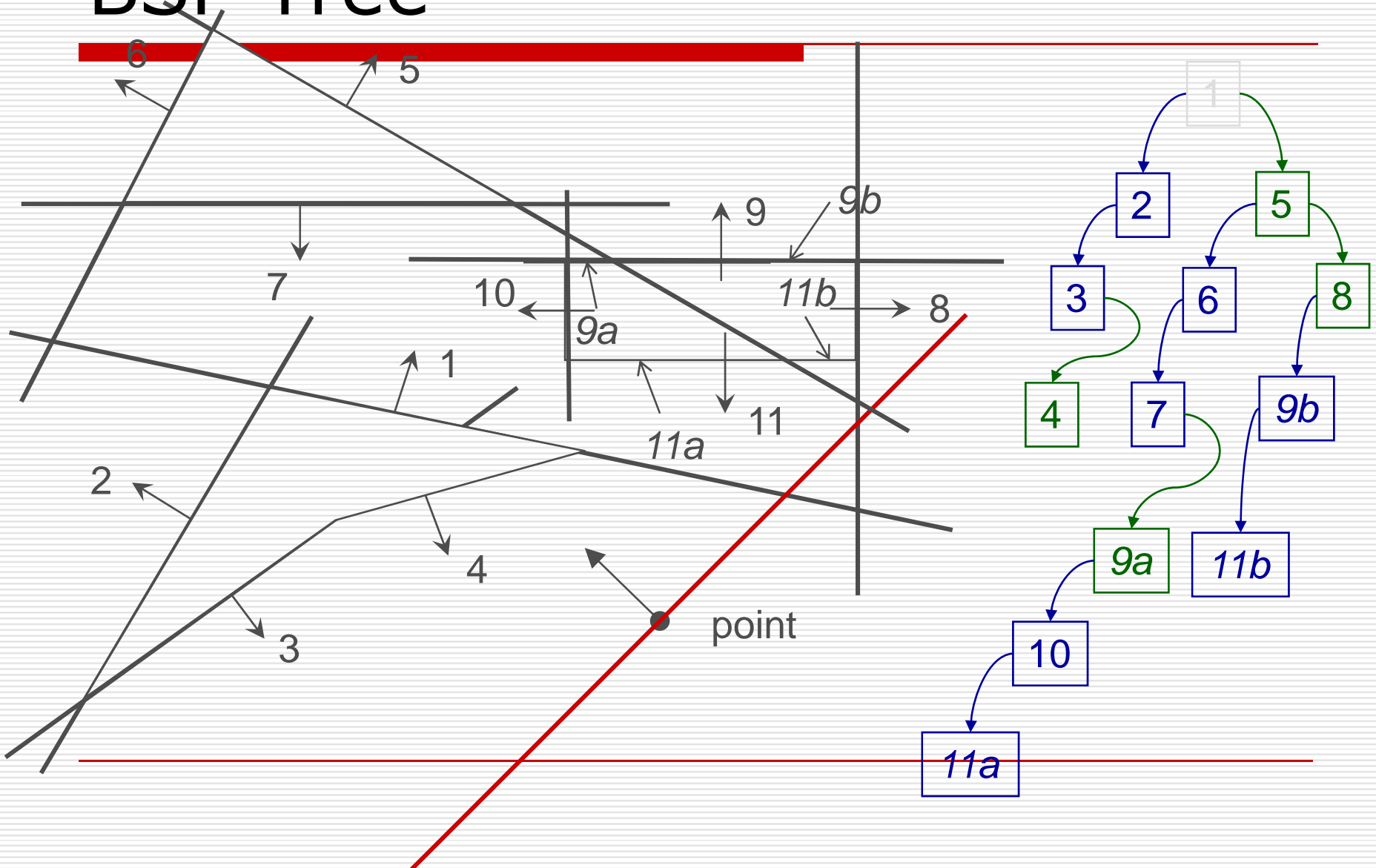
BSP Tree



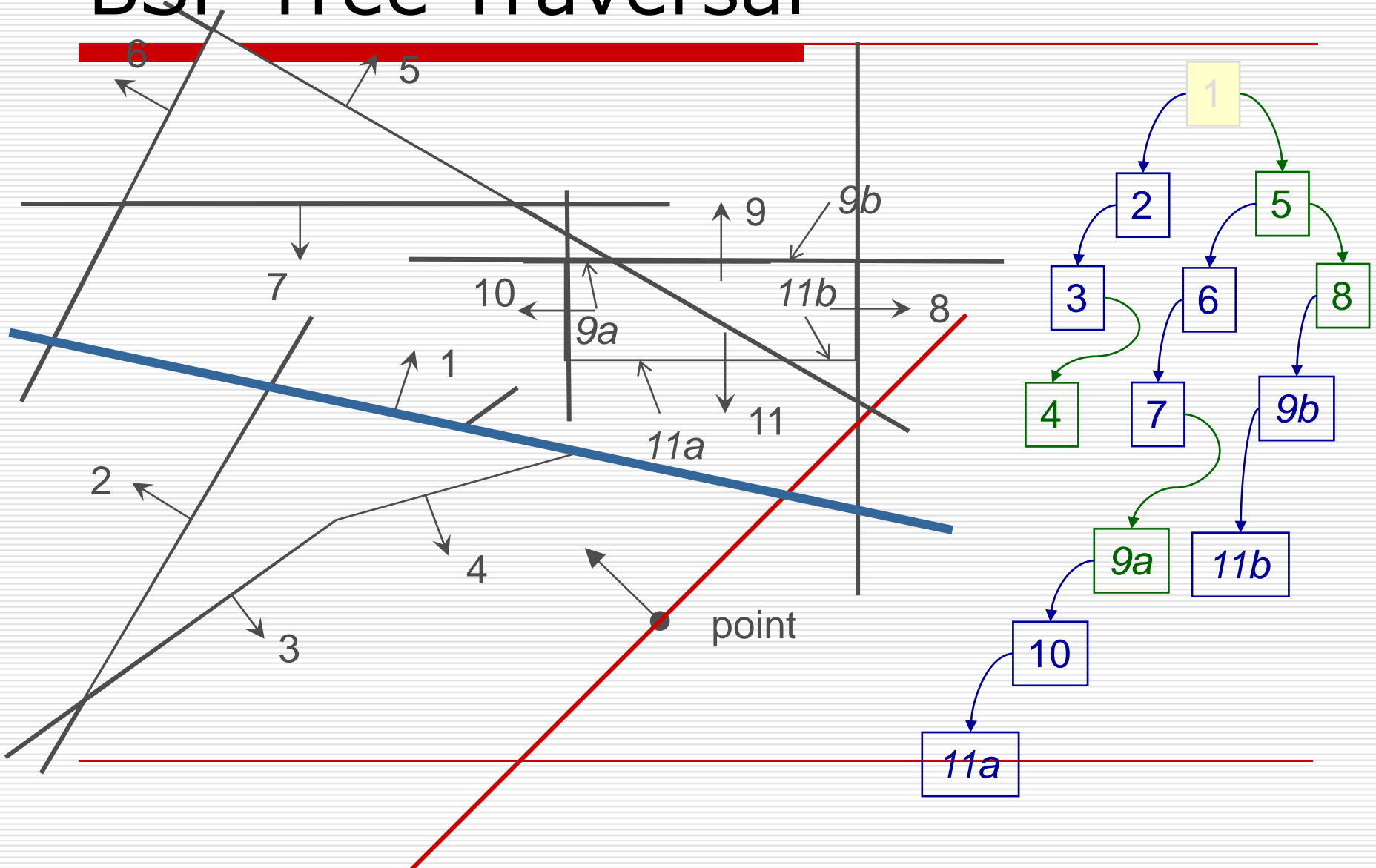
BSP Tree



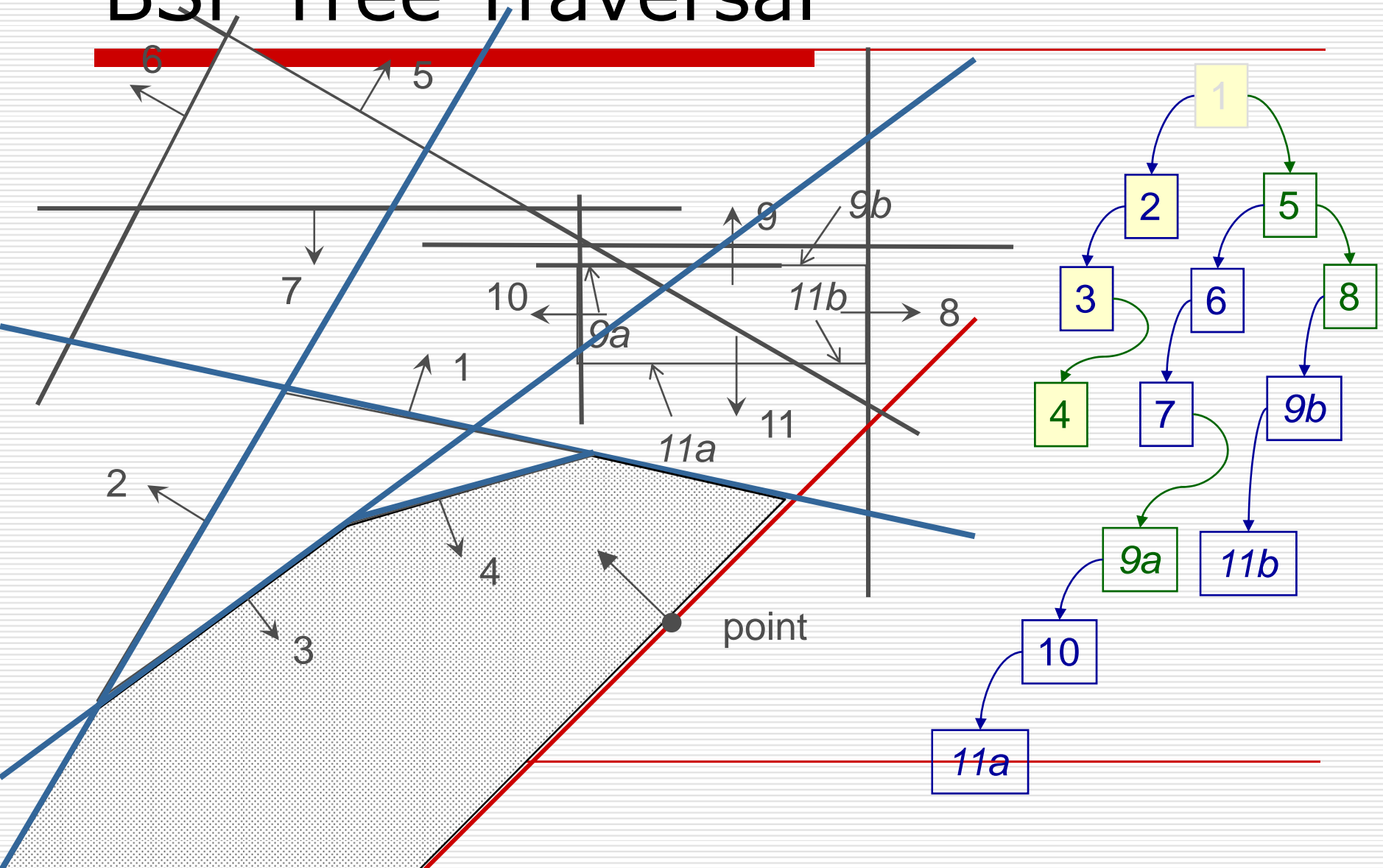
BSP Tree



BSP Tree Traversal



BSP Tree Traversal



The z-Buffer Algorithm

- Resolve depths at the pixel level
 - Idea: add Z to frame buffer, when a pixel is drawn, check whether it is closer than what's already in the frame buffer
-

The z-Buffer Algorithm

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

+

5	5	5	5	5	5	5
5	5	5	5	5	5	
5	5	5	5	5		
5	5	5	5			
5	5	5				
5	5					
5						

=

5	5	5	5	5	5	5	0
5	5	5	5	5	5	0	0
5	5	5	5	5	0	0	0
5	5	5	5	0	0	0	0
5	5	5	0	0	0	0	0
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5	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

5	5	5	5	5	5	5	0
5	5	5	5	5	5	0	0
5	5	5	5	5	0	0	0
5	5	5	5	0	0	0	0
5	5	5	0	0	0	0	0
5	5	0	0	0	0	0	0
5	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

+

3					
4	3				
5	4	3			
6	5	4	3		
7	6	5	4	3	
8	7	6	5	4	3

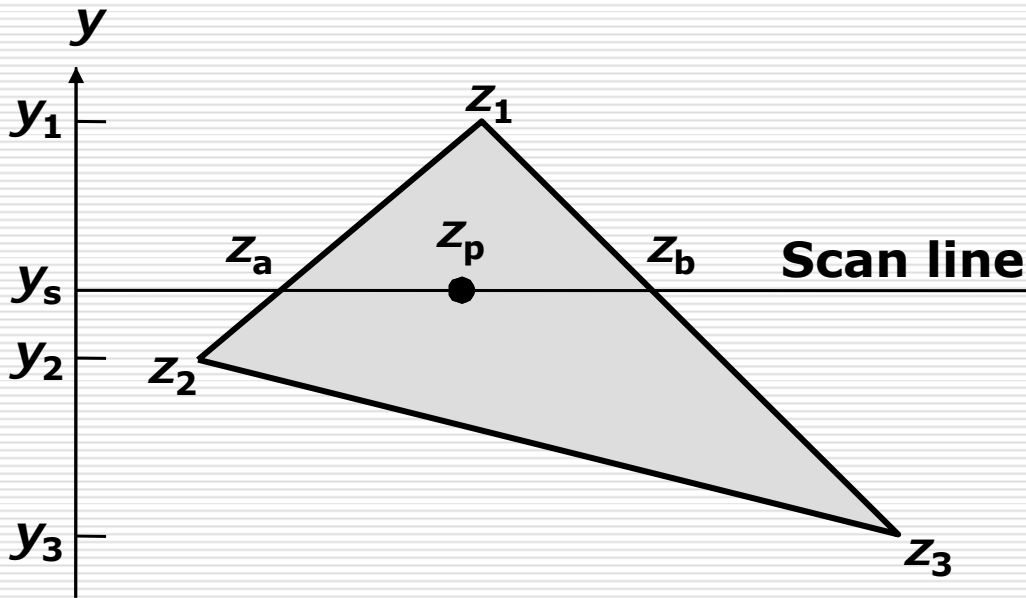
=

5	5	5	5	5	5	5	0
5	5	5	5	5	5	0	0
5	5	5	5	5	0	0	0
5	5	5	5	0	0	0	0
6	5	5	3	0	0	0	0
7	6	5	4	3	0	0	0
8	7	6	5	4	3	0	0
0	0	0	0	0	0	0	0

The z-Buffer Algorithm

```
void zBuffer() {  
    int pz;  
    for (each polygon) {  
        for (each pixel in polygon's projection) {  
            pz = polygon's z-value at (x,y);  
            if (pz >= ReadZ(x,y)) {  
                WriteZ(x,y,pz);  
                WritePixel(x,y,color);  
            }  
        }  
    }  
}
```

The z-Buffer Algorithm



$$z_a = z_1 - (z_1 - z_2) \frac{y_1 - y_s}{y_1 - y_2}$$

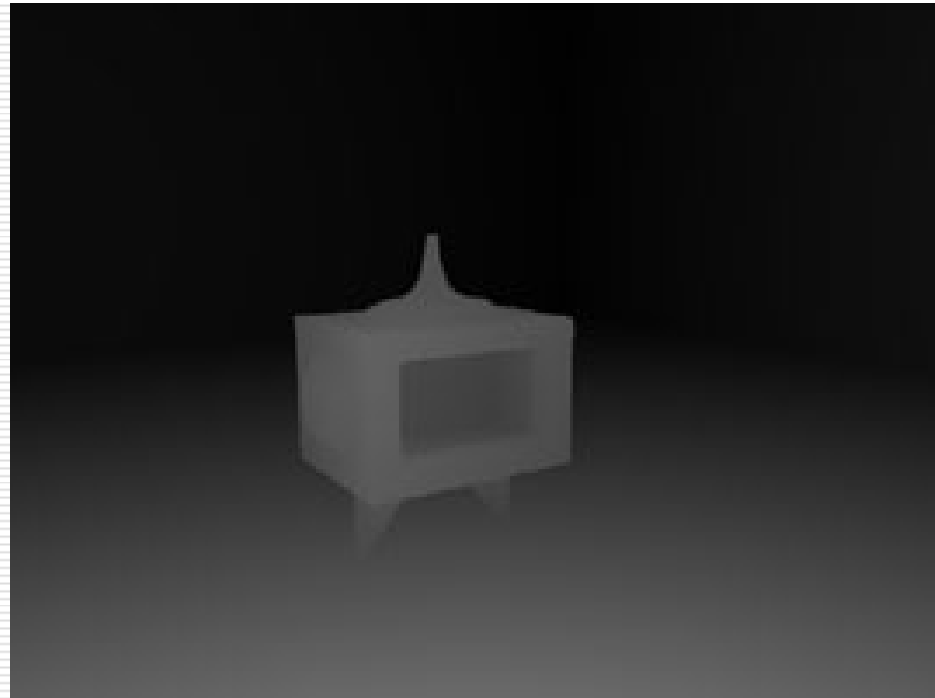
$$z_b = z_1 - (z_1 - z_3) \frac{y_1 - y_s}{y_1 - y_3}$$

$$z_p = z_b - (z_b - z_a) \frac{x_b - x_p}{x_b - x_a}$$

z-Buffer: Example



color buffer



depth buffer

The z-Buffer Algorithm

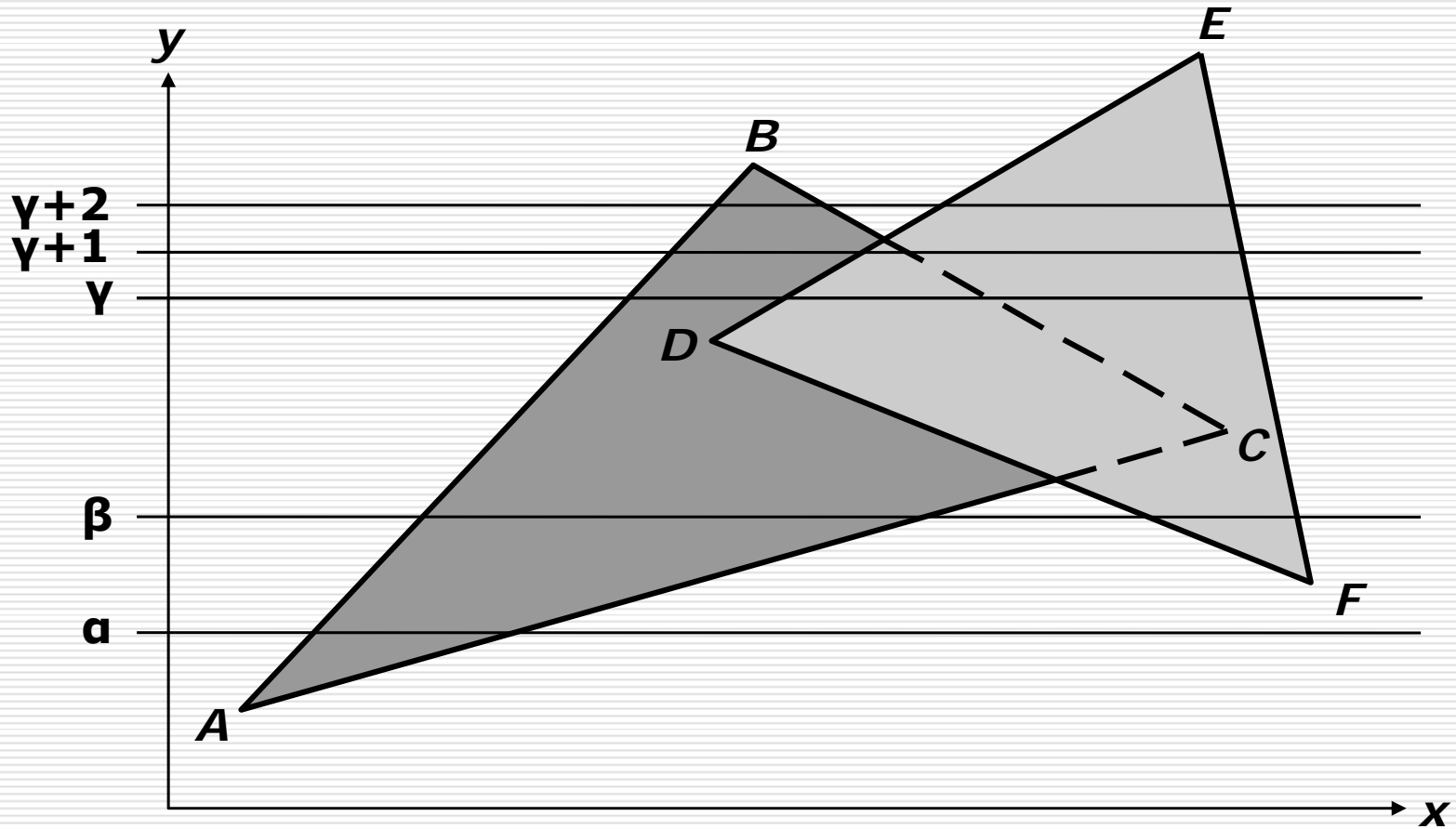
□ Benefits

- Easy to implement
- Works for any geometric primitive
- Parallel operation in hardware
 - independent of order of polygon drawn

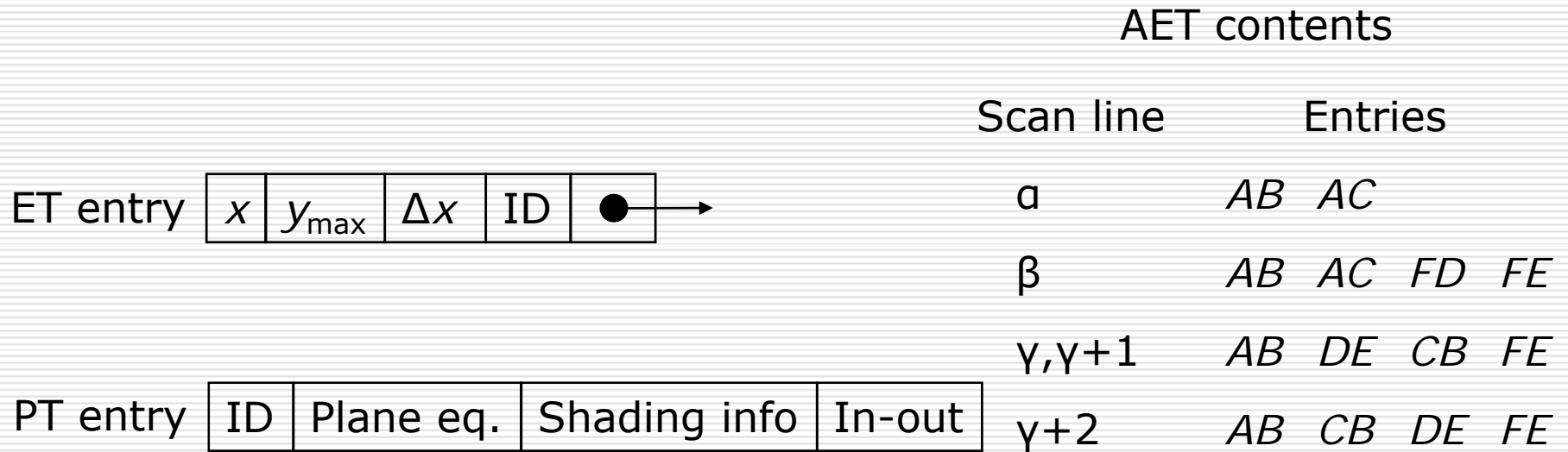
□ Limitations

- Memory required for depth buffer
 - Quantization and aliasing artifacts
 - Overflow
 - Transparency does not work well
-

Scan-Line Algorithm



Scan-Line Algorithm



- ET = edge table
 - PT = polygon table
 - AET = active-edge table
-

General Scan-Line Algorithm

add surfaces to polygon table (PT);
initialize active-edge table (AET);

for (each scan line) {
 update AET;

for (each pixel on scan line) {
 determine surfaces in AET that project to pixel;
 find closest such surface;
 determine closest surface's shade at pixel;

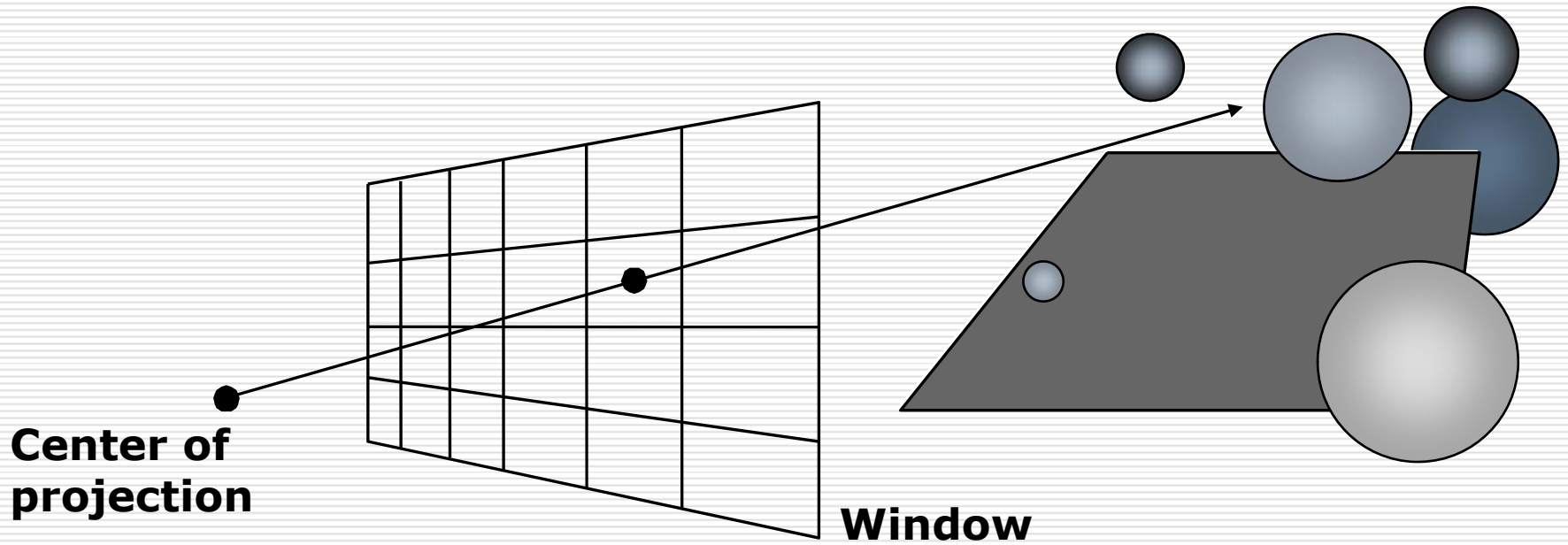
 }

 }

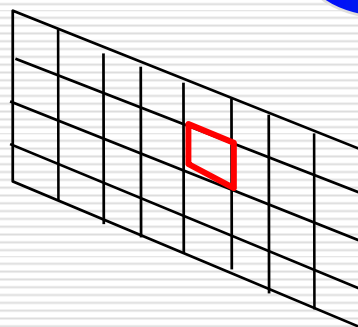
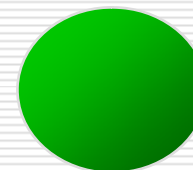
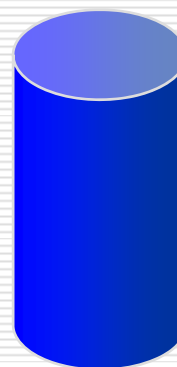
Ray Tracing = Ray Casting

```
select center of projection and window on viewplane;
for (each scan line in image) {
    for (each pixel in scan line) {
        determine ray from center of projection through pixel;
        for (each object in scene) {
            if (object is intersected and is closest considered thus far)
                record intersection and object name;
        }
        set pixel's color to that at closest object intersection;
    }
}
```

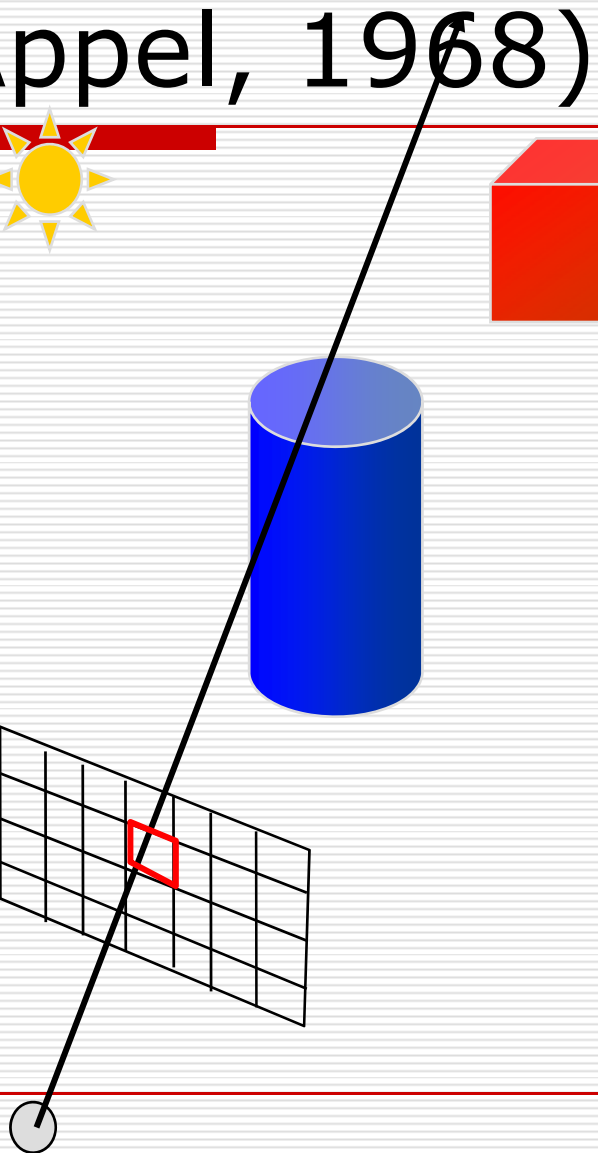
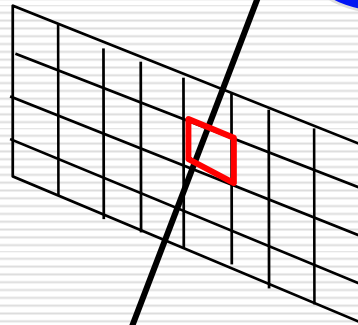
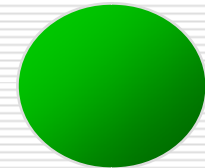
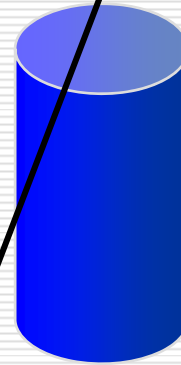
Ray Casting



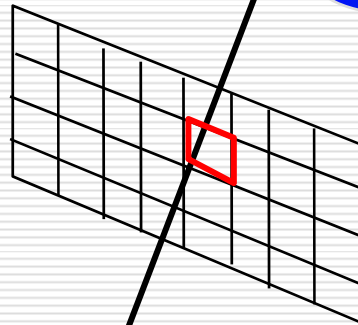
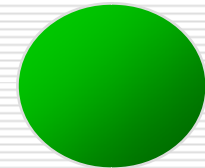
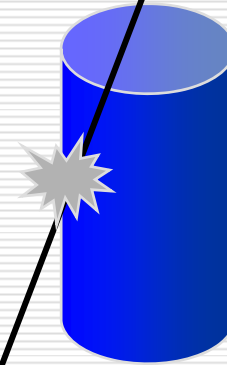
Ray Casting (Appel, 1968)



Ray Casting (Appel, 1968)

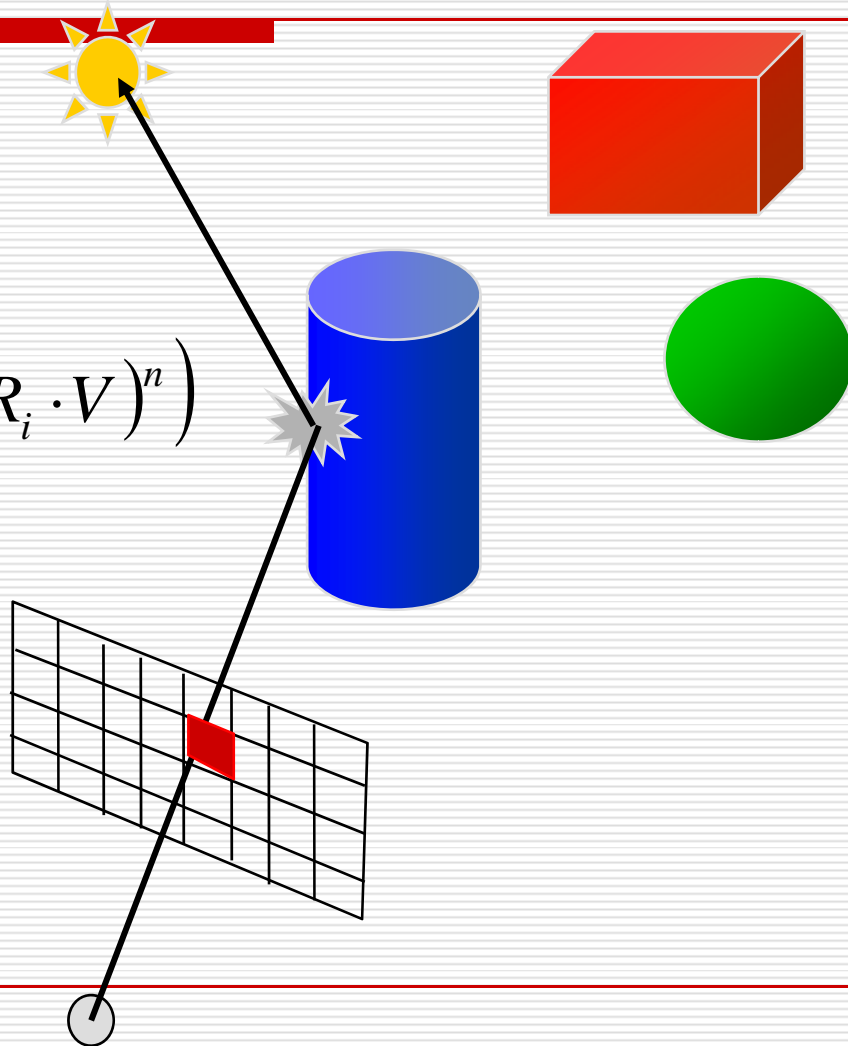


Ray Casting (Appel, 1968)

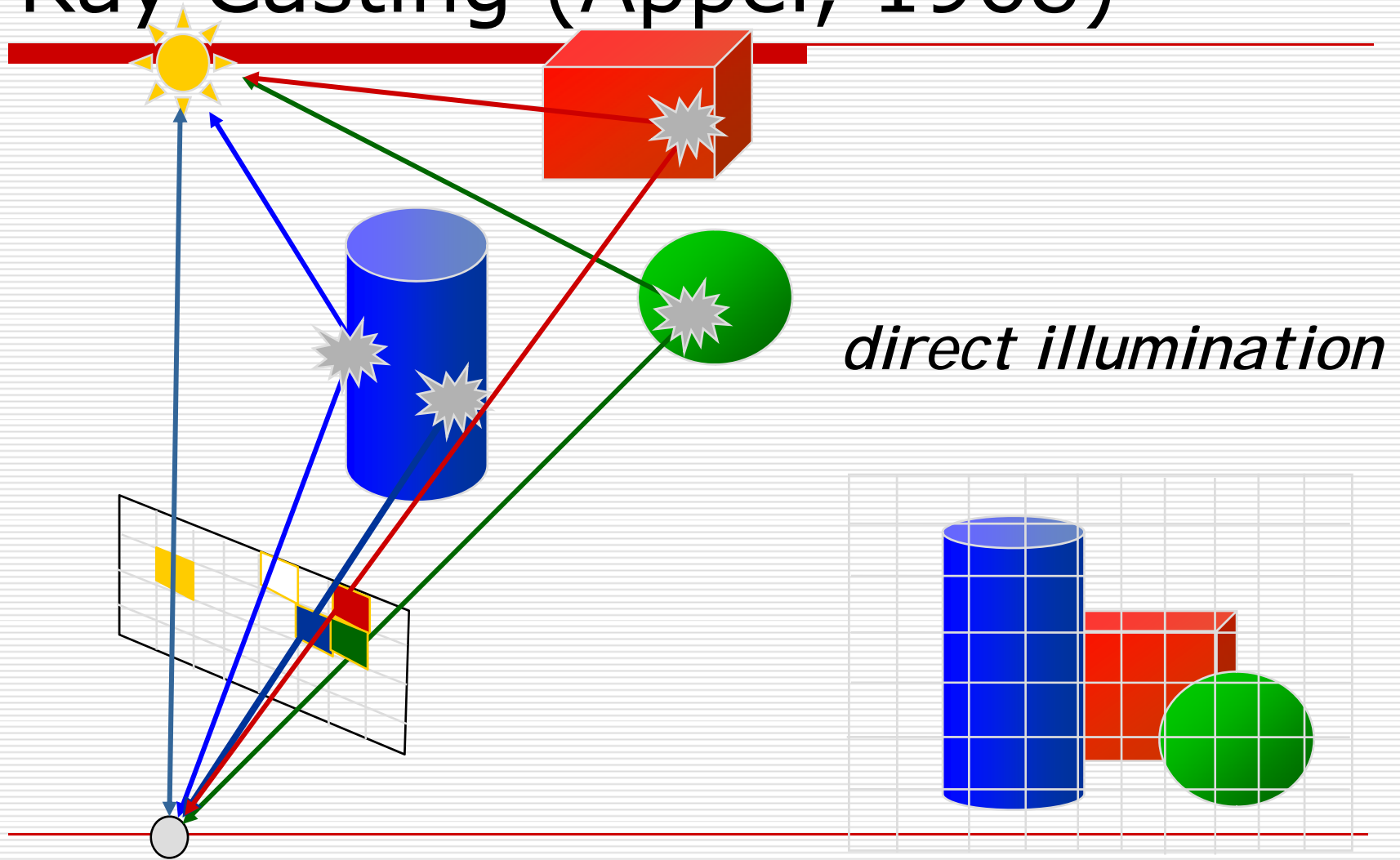


Ray Casting (Appel, 1968)

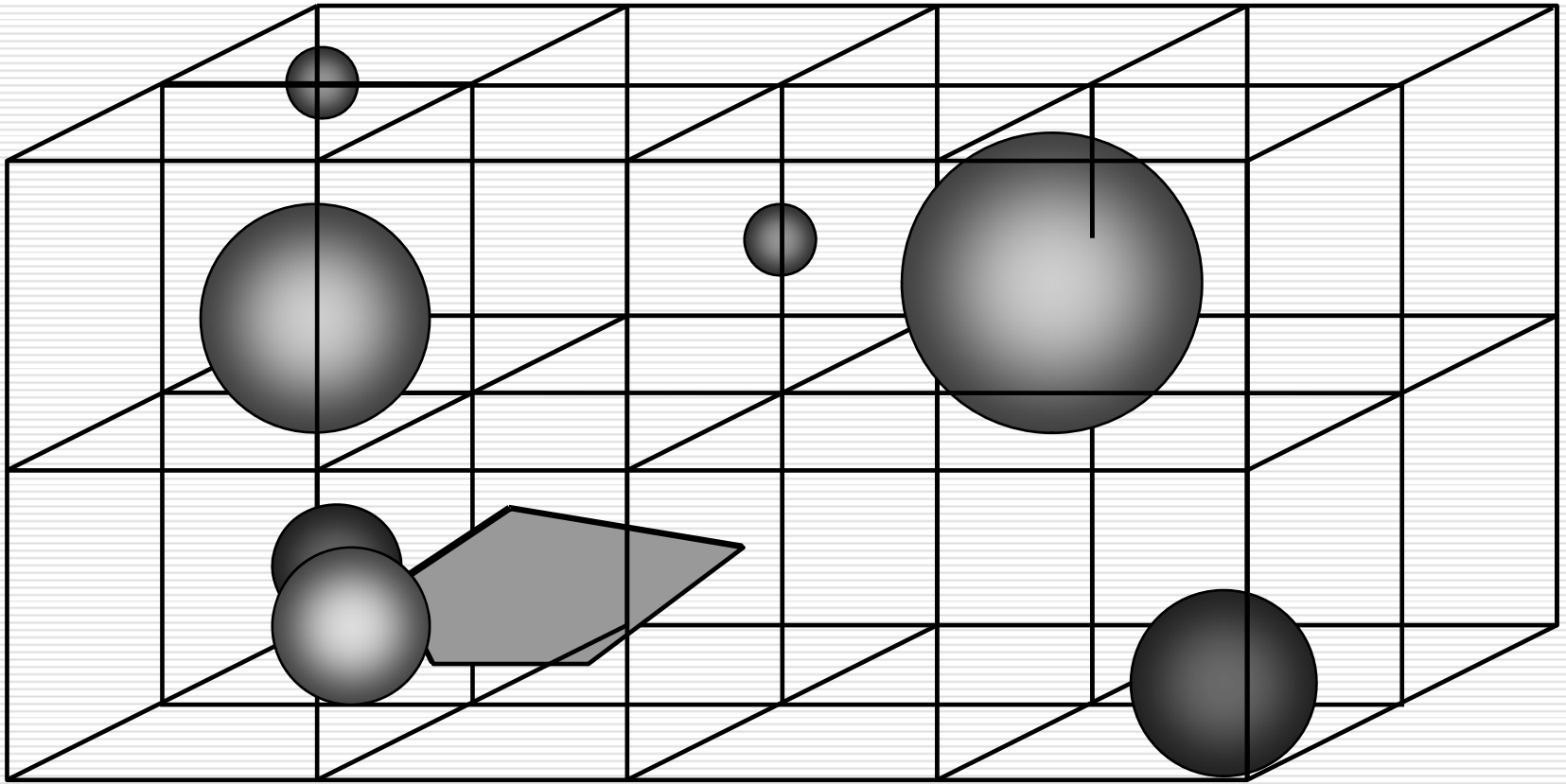
$$k_a I_a + \sum_{i=1}^{nls} I_i \left(k_d (L_i \cdot N) + k_s (R_i \cdot V)^n \right)$$



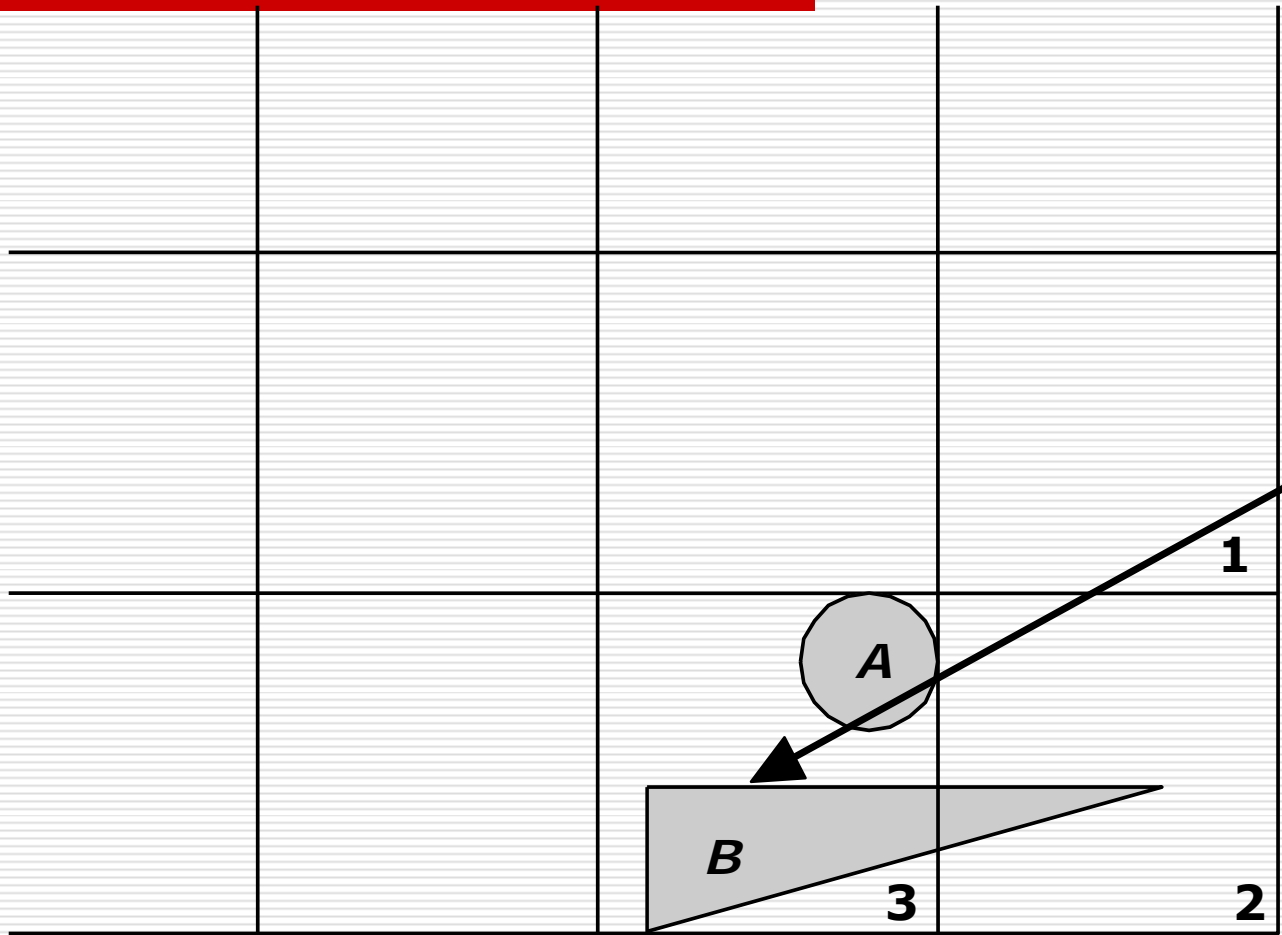
Ray Casting (Appel, 1968)



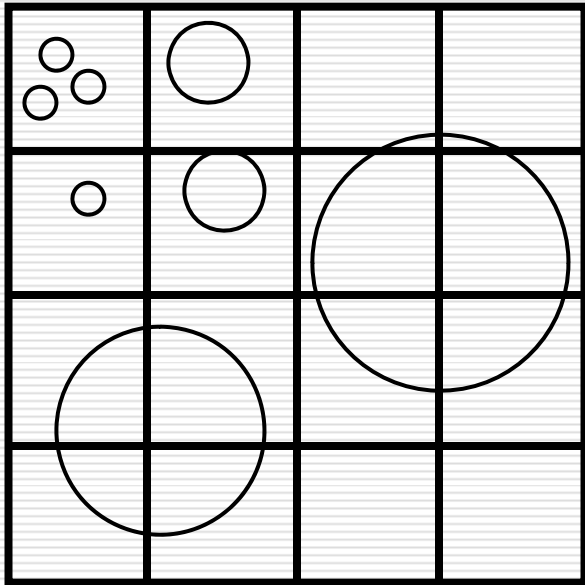
Spatial Partitioning



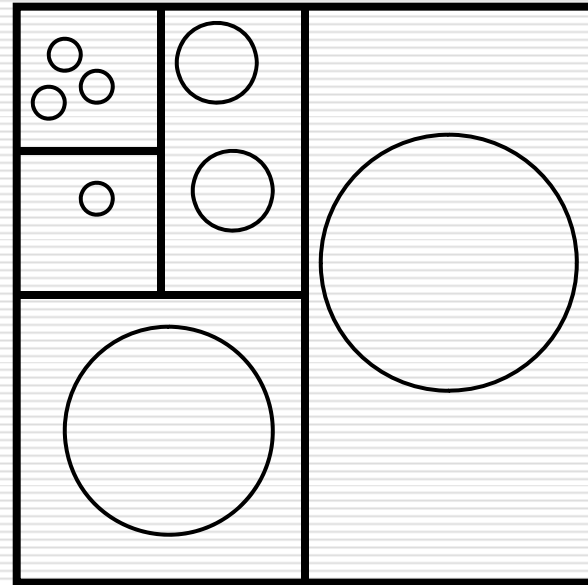
Spatial Partitioning



Space Subdivision Approaches

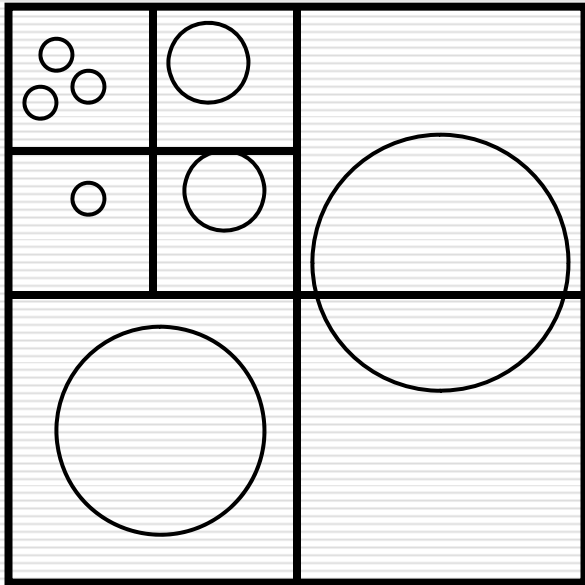


Uniform grid

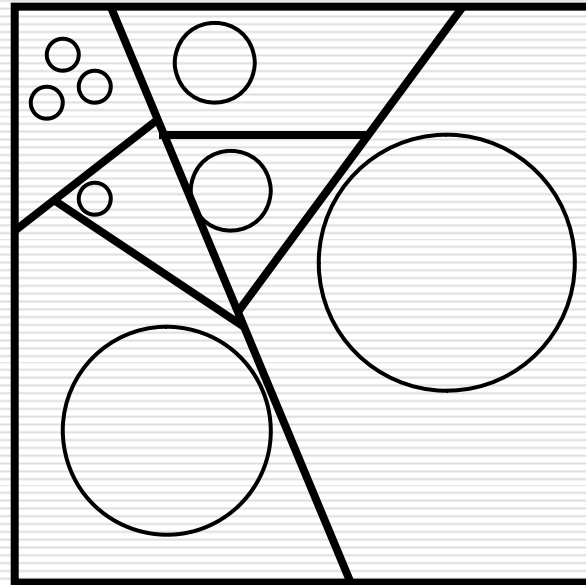


K-d tree

Space Subdivision Approaches

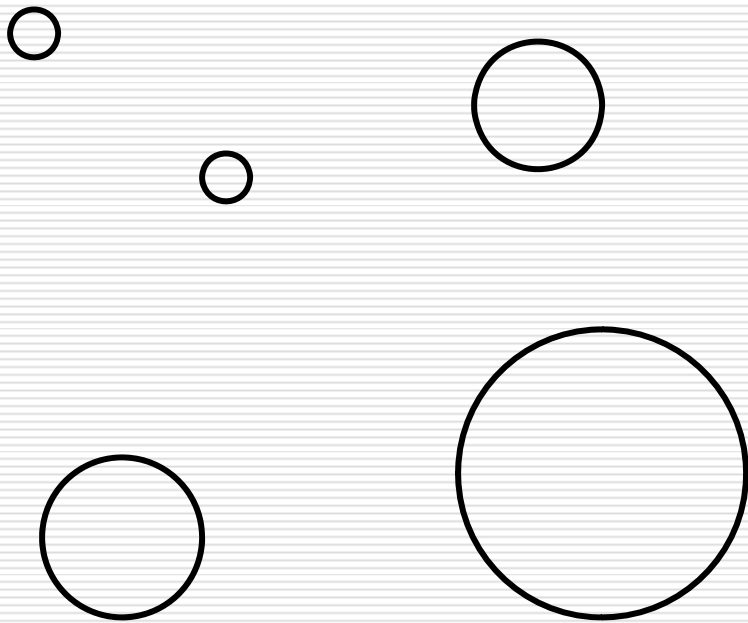


Quadtree (2D)
Octree (3D)

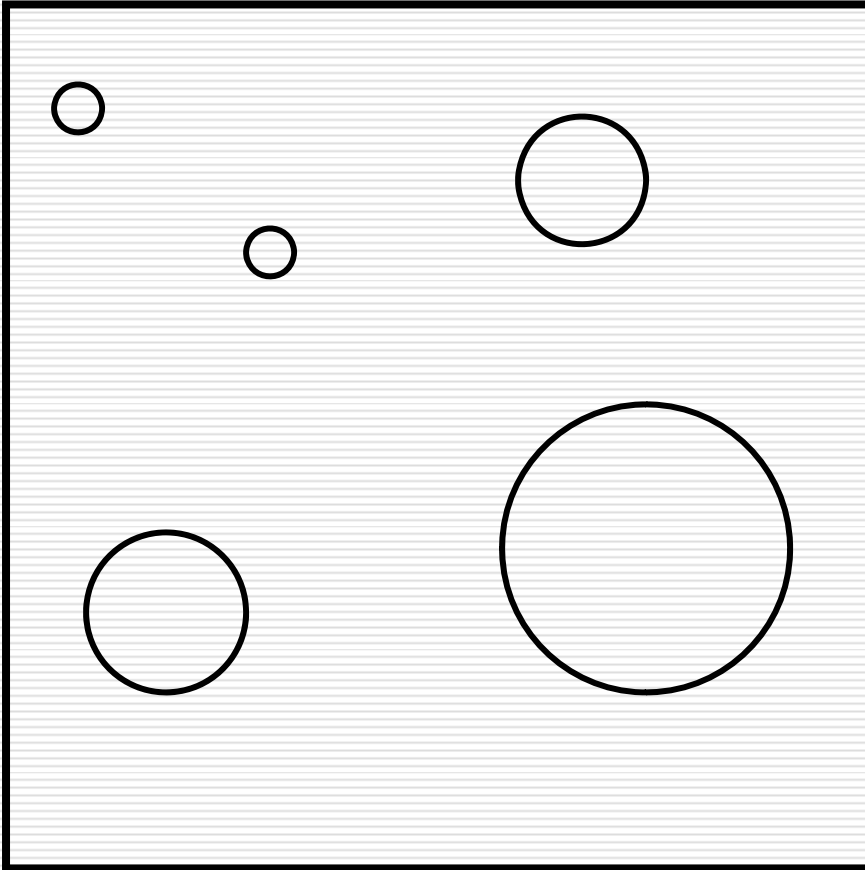


BSP tree

Uniform Grid



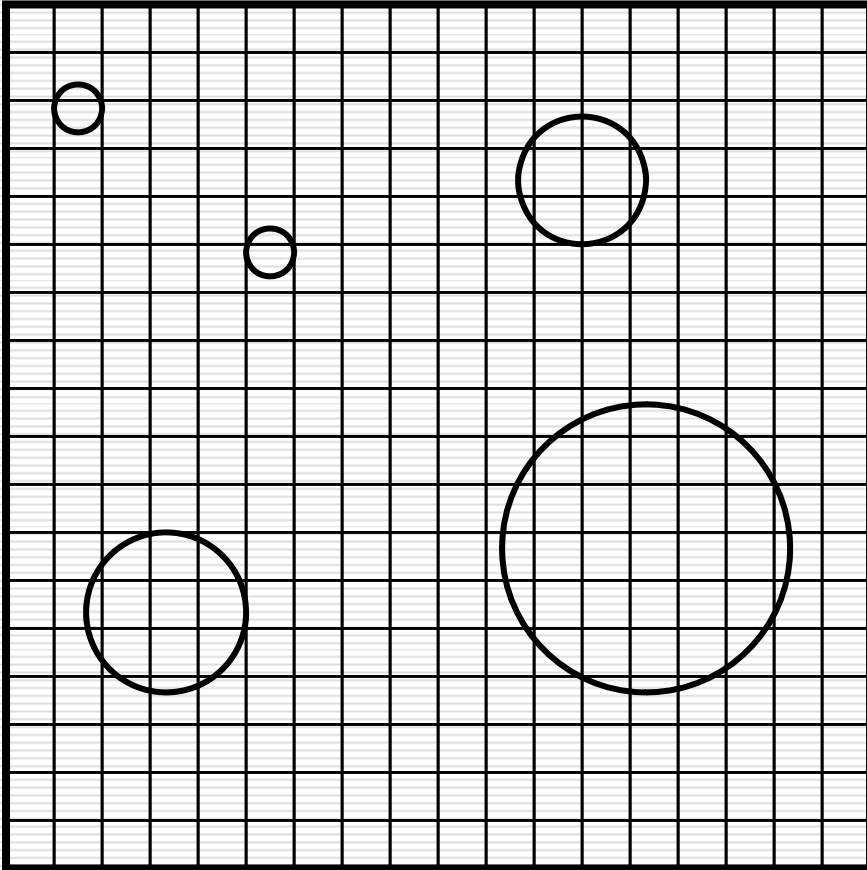
Uniform Grid



Preprocess scene

1. Find bounding box
-

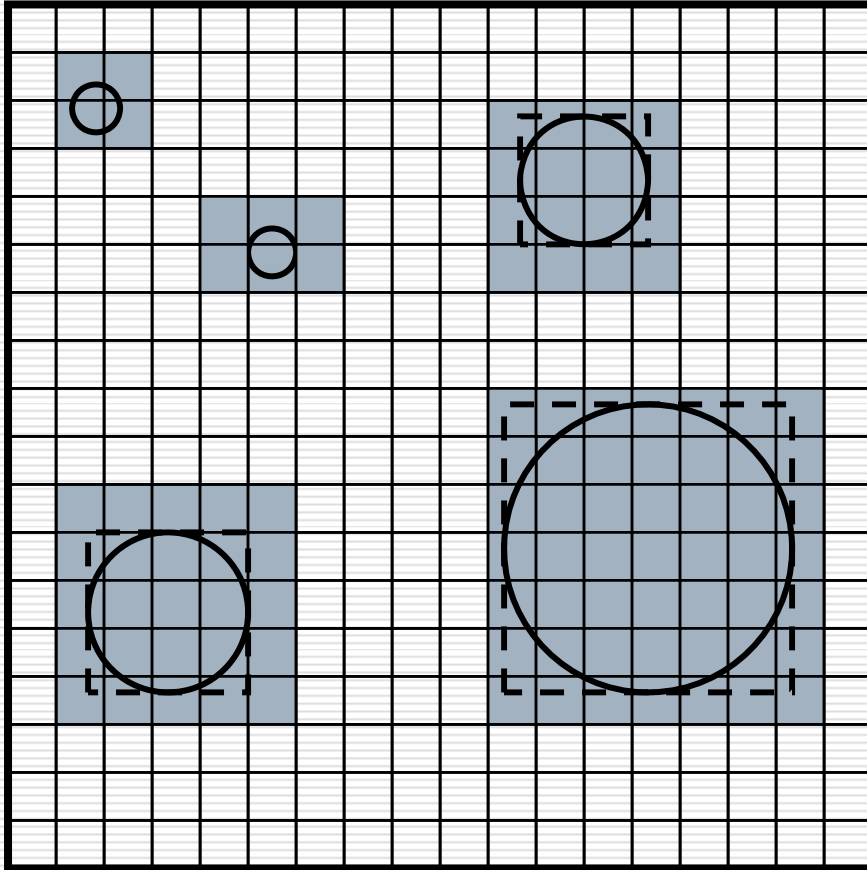
Uniform Grid



Preprocess scene

1. Find bounding box
 2. Determine grid resolution
-

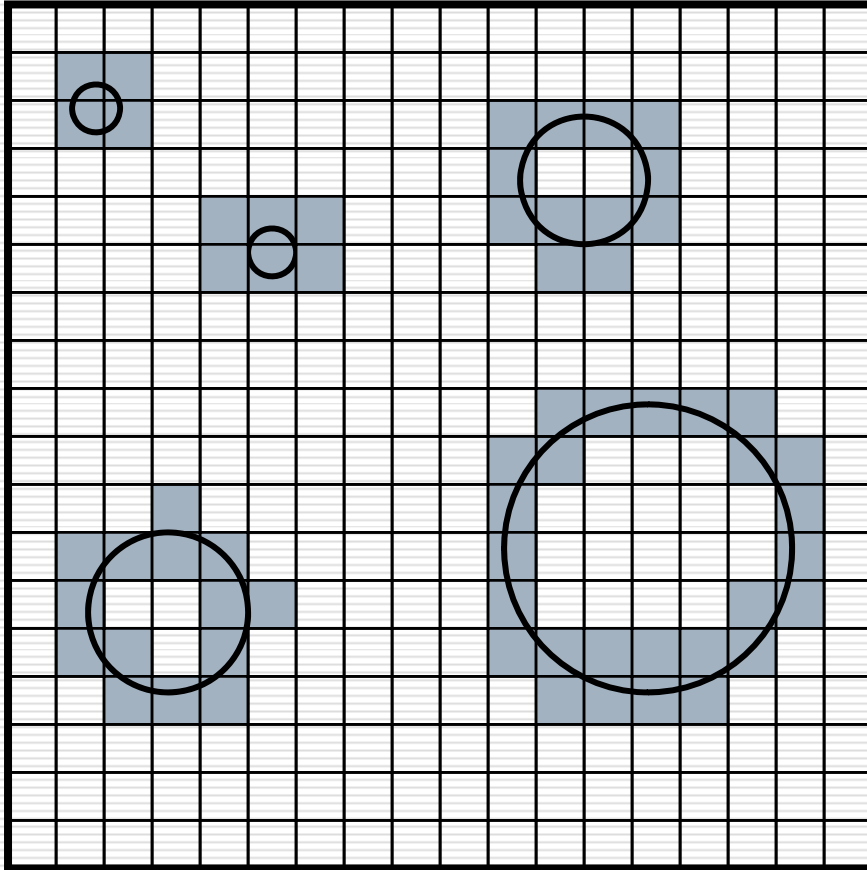
Uniform Grid



Preprocess scene

1. Find bounding box
2. Determine grid resolution
3. Place object in cell if its bounding box overlaps the cell

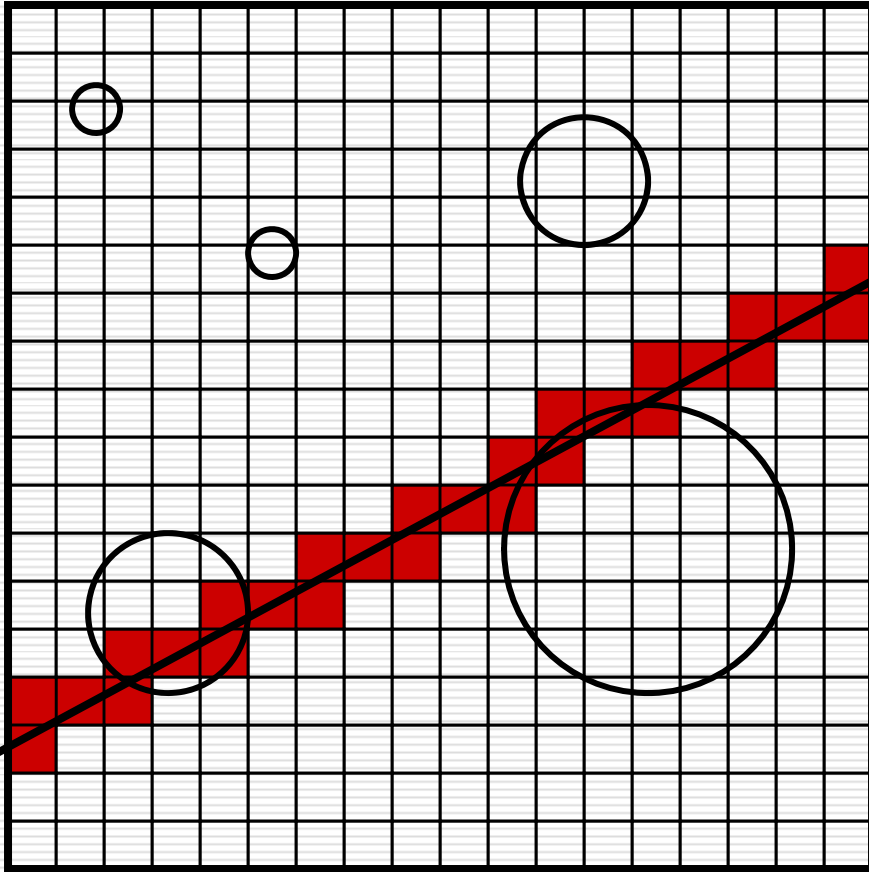
Uniform Grid



Preprocess scene

1. Find bounding box
 2. Determine grid resolution
 3. Place object in cell if its bounding box overlaps the cell
 4. Check that object overlaps cell (expensive!)
-

Uniform Grid Traversal

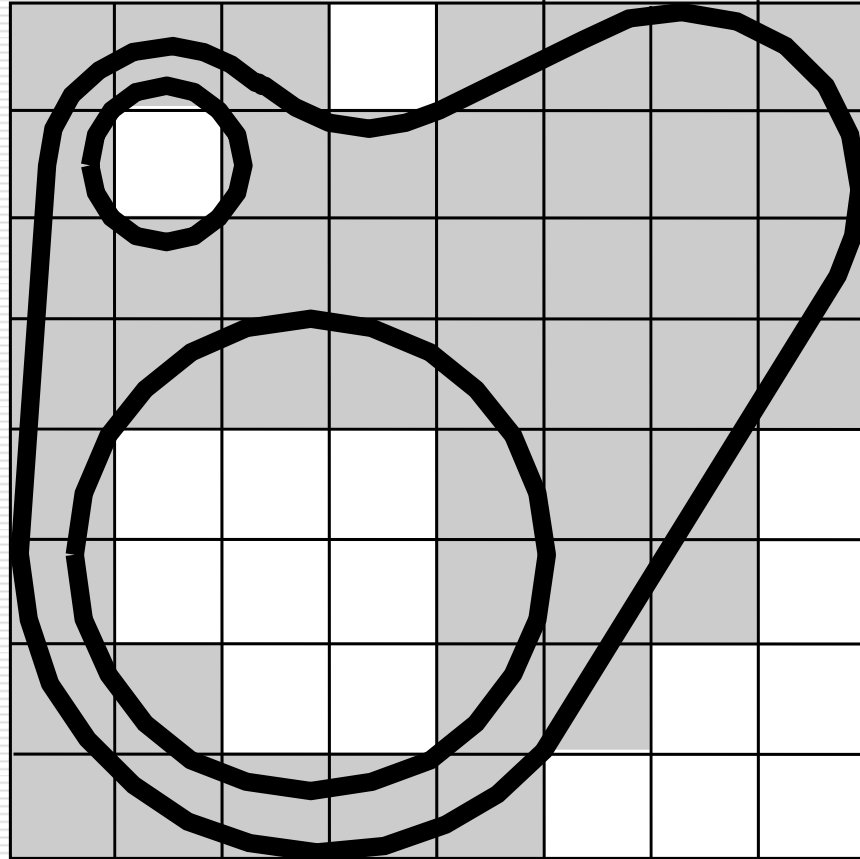


Preprocess scene

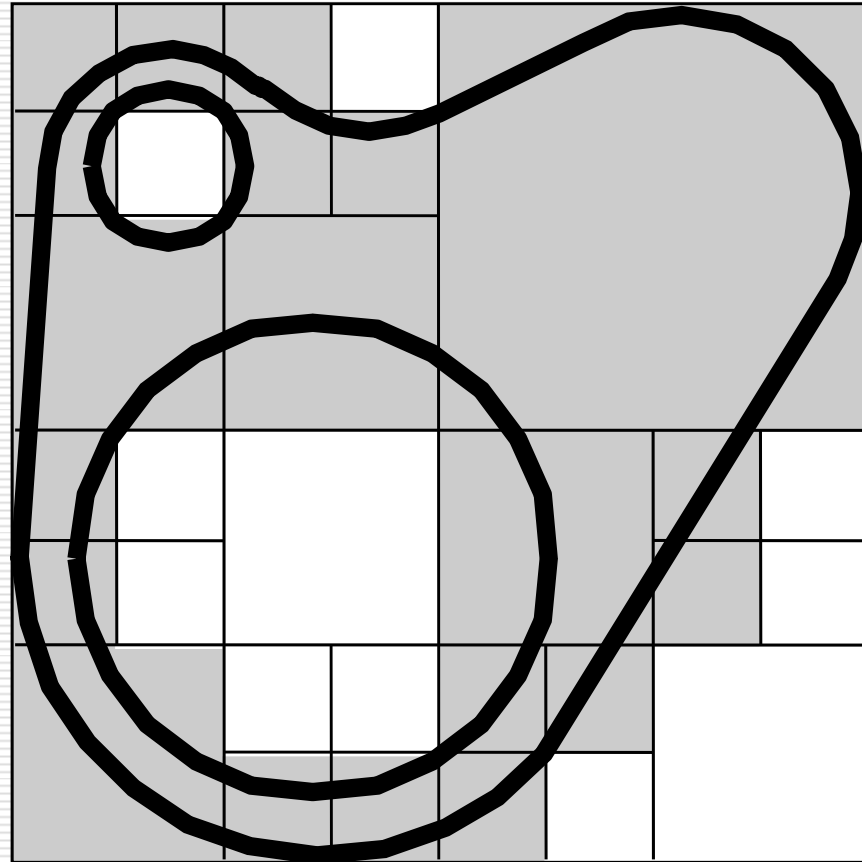
Traverse grid

3D line = 3D-DDA

From Uniform Grid to Quadtree

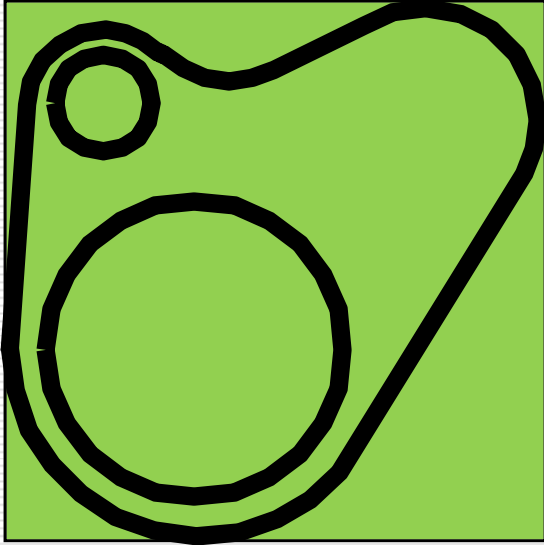


Quadtree (Octrees)



subdivide the space adaptively

Quadtree Data Structure

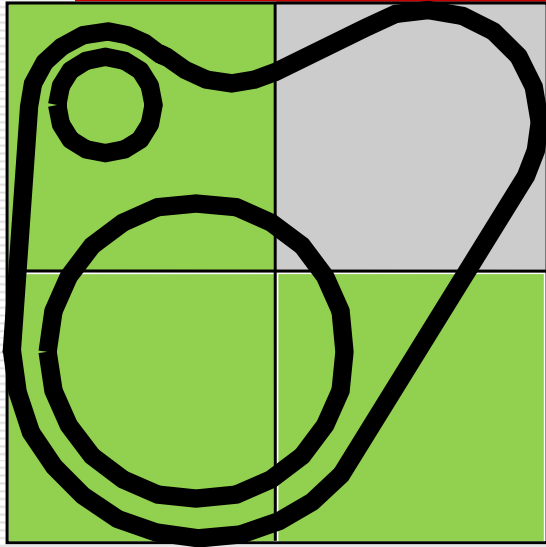


2	3
0	1

Quadrant Numbering

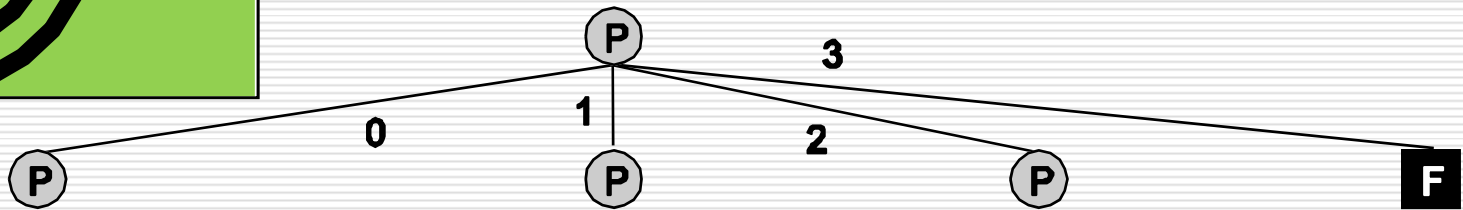
P

Quadtree Data Structure

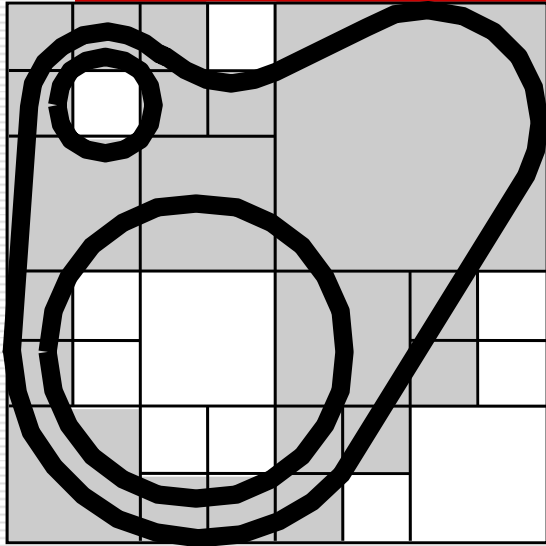


2	3
0	1

Quadrant Numbering

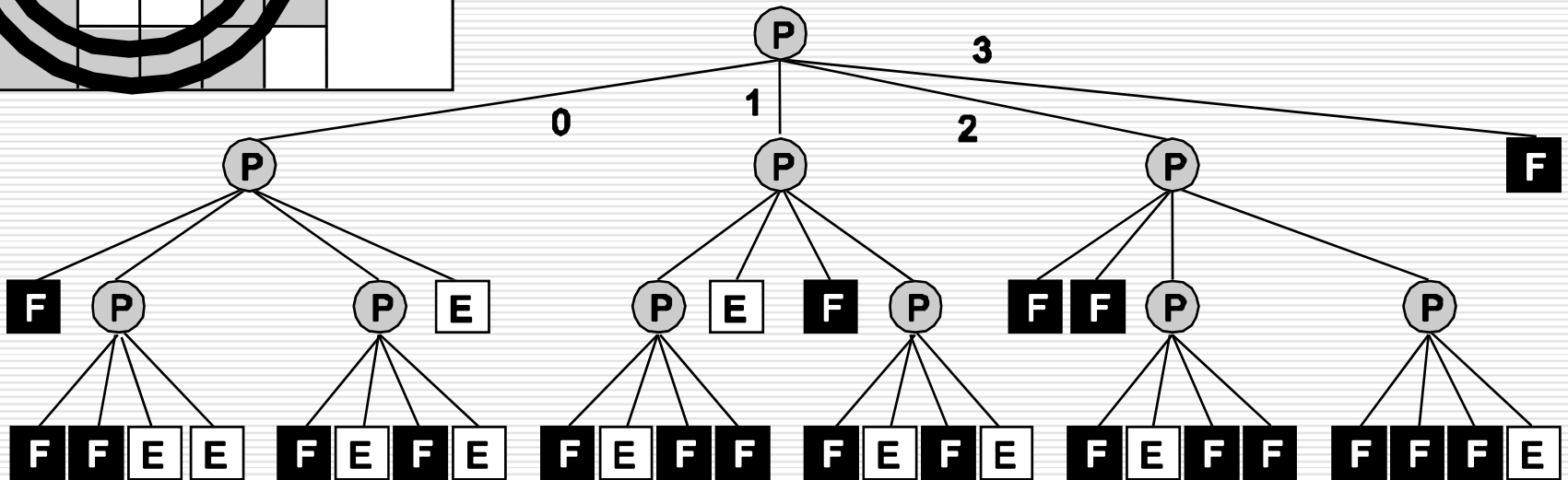


Quadtree Data Structure

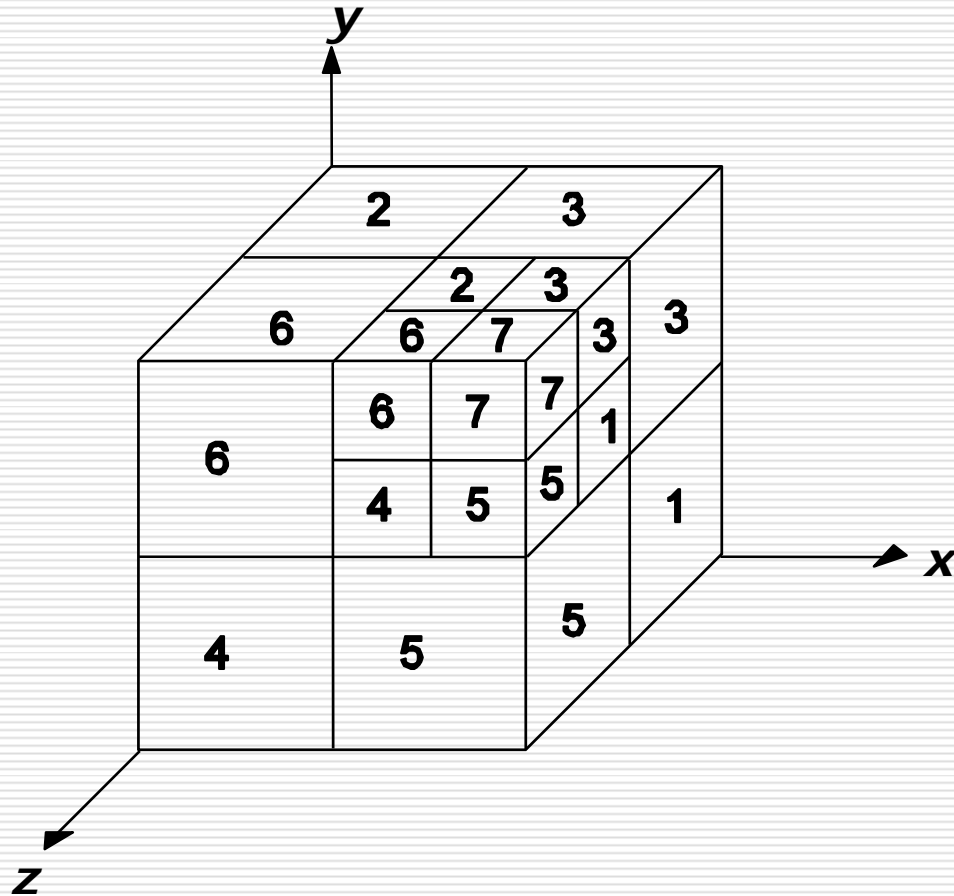


2	3
0	1

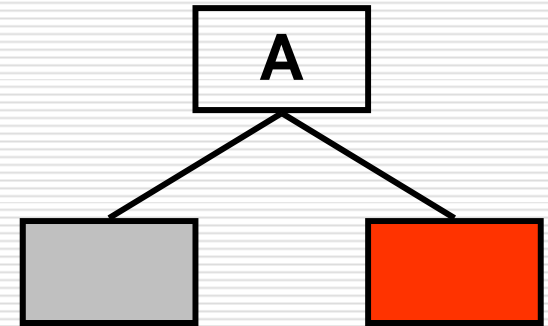
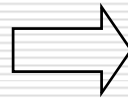
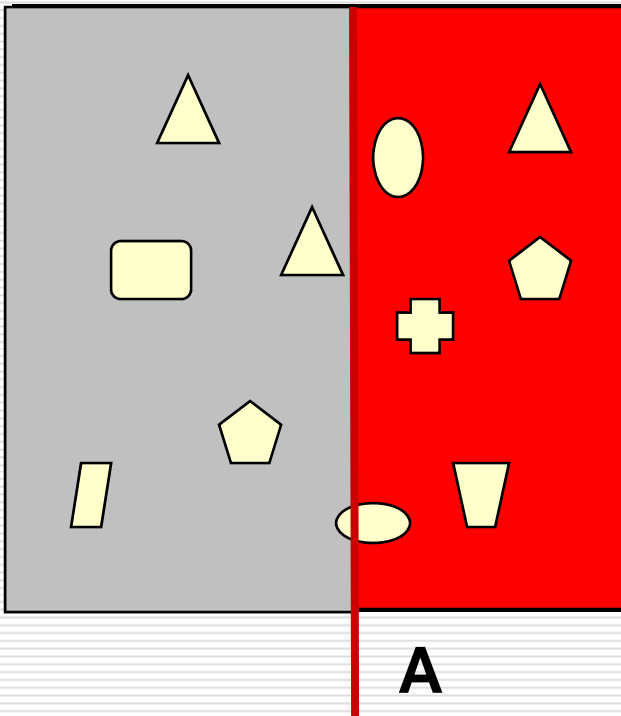
Quadrant Numbering



From Quadtree to Octree

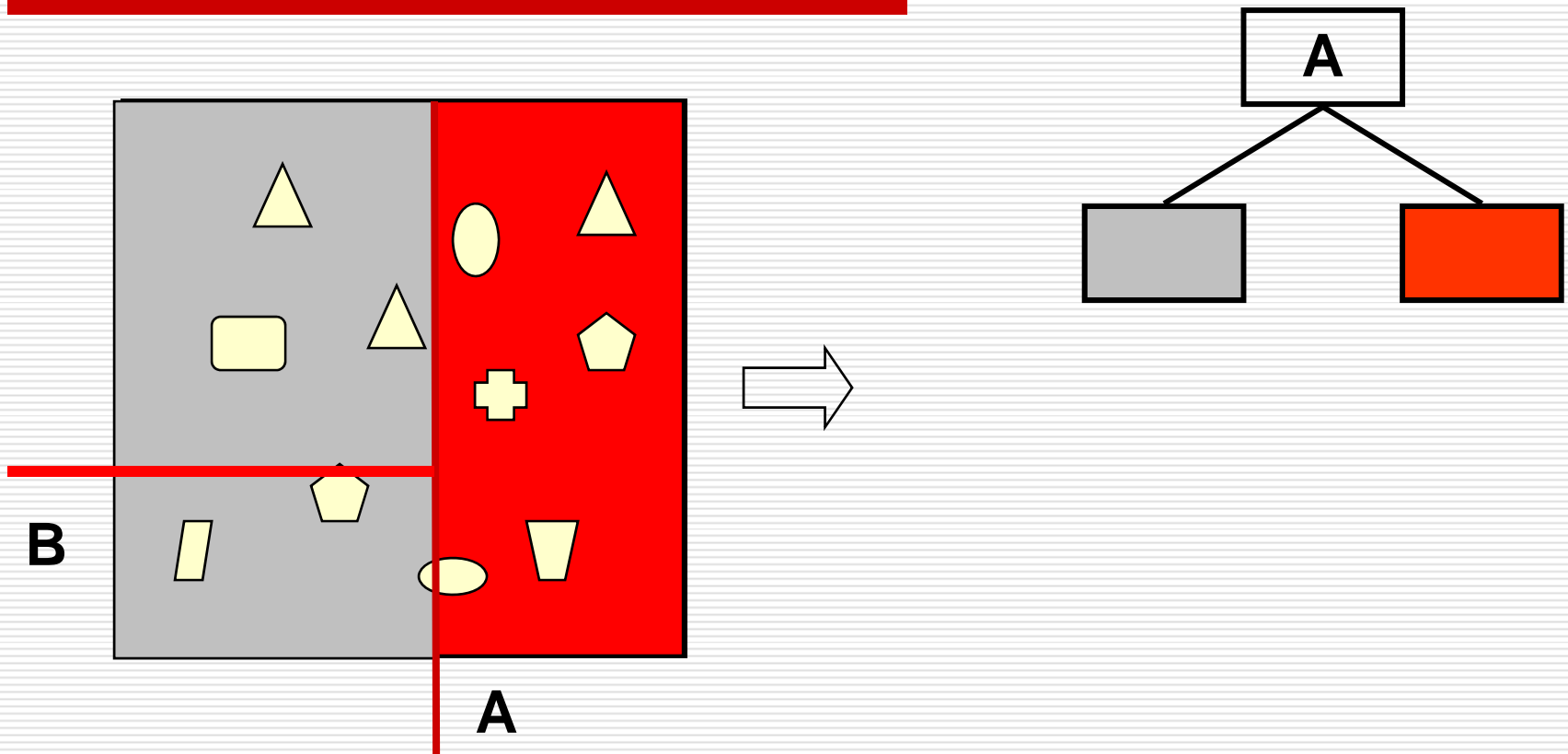


K-d Tree



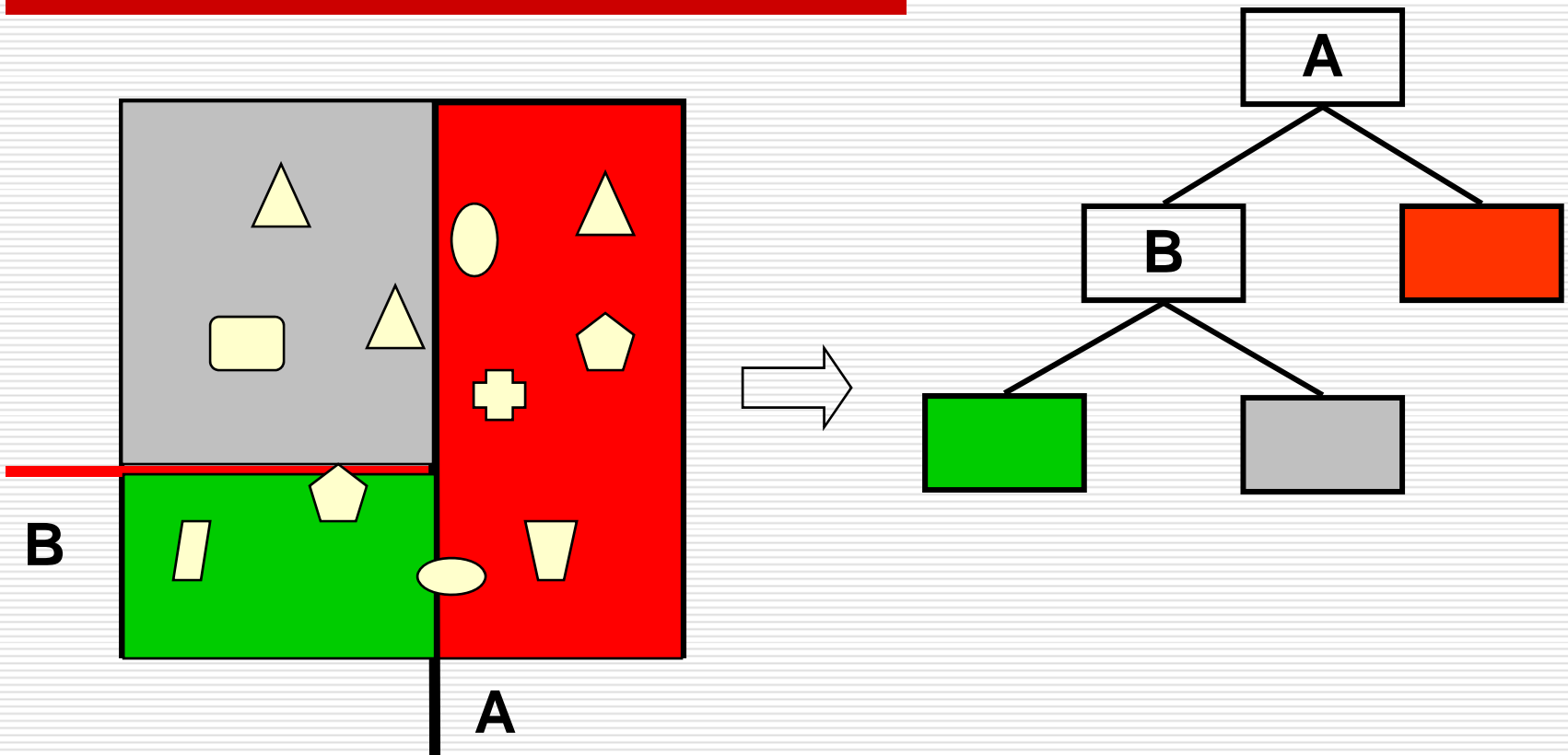
Leaf nodes correspond to unique regions in space

K-d Tree



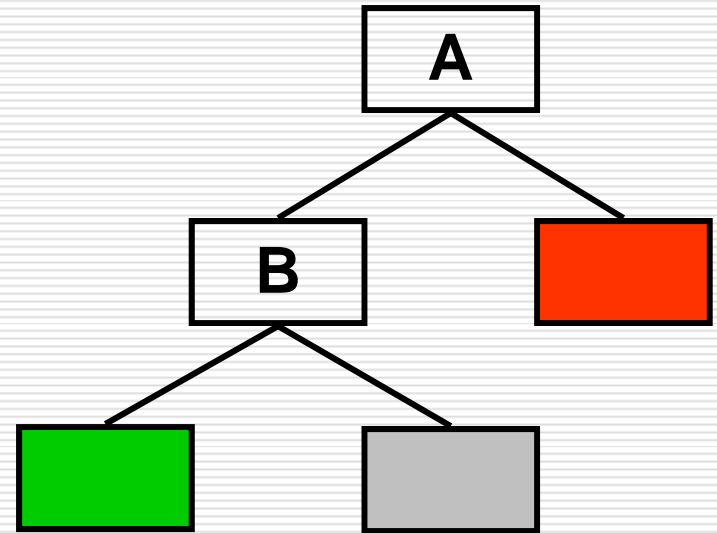
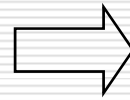
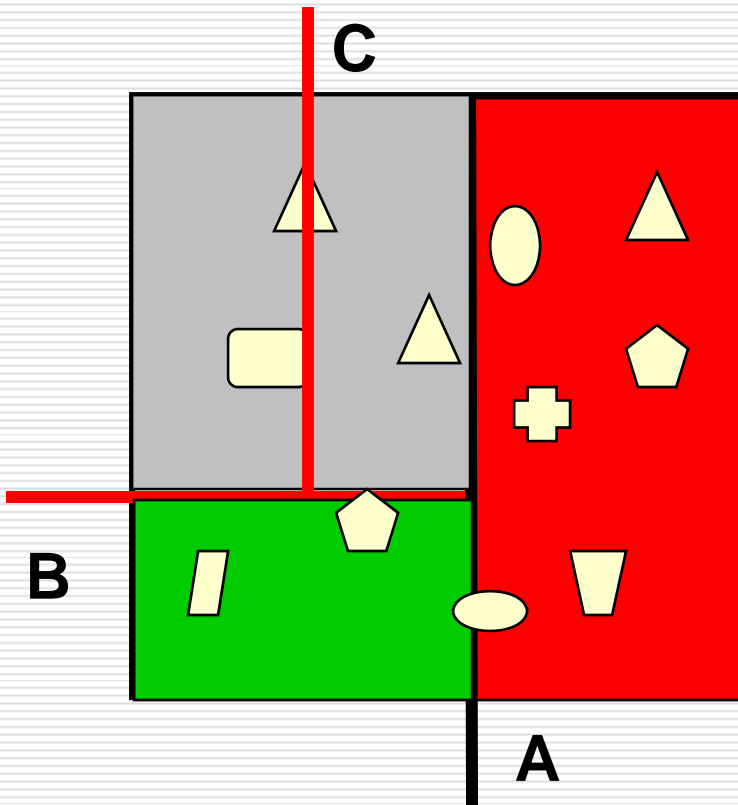
Leaf nodes correspond to unique regions in space

K-d Tree

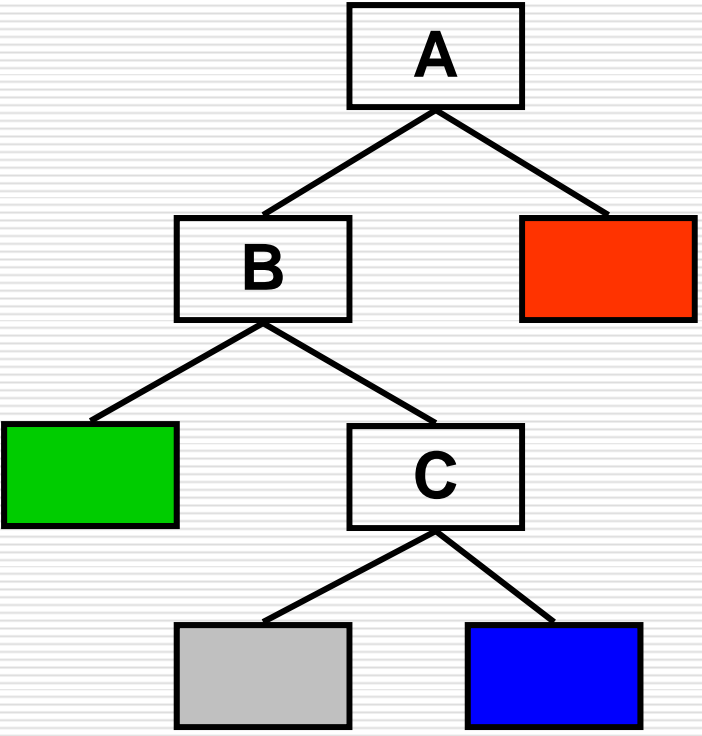
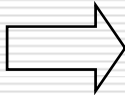
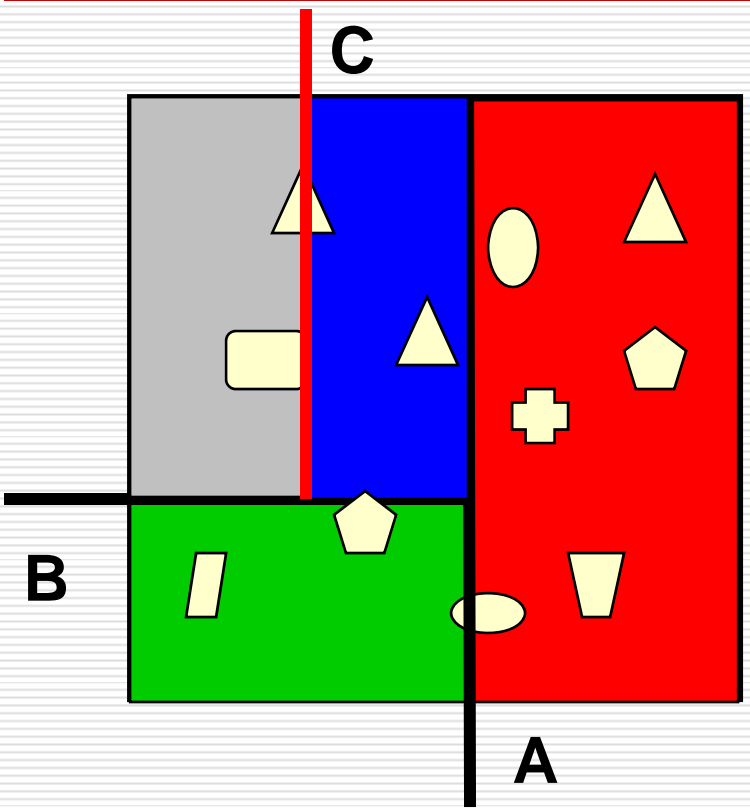


Leaf nodes correspond to unique regions in space

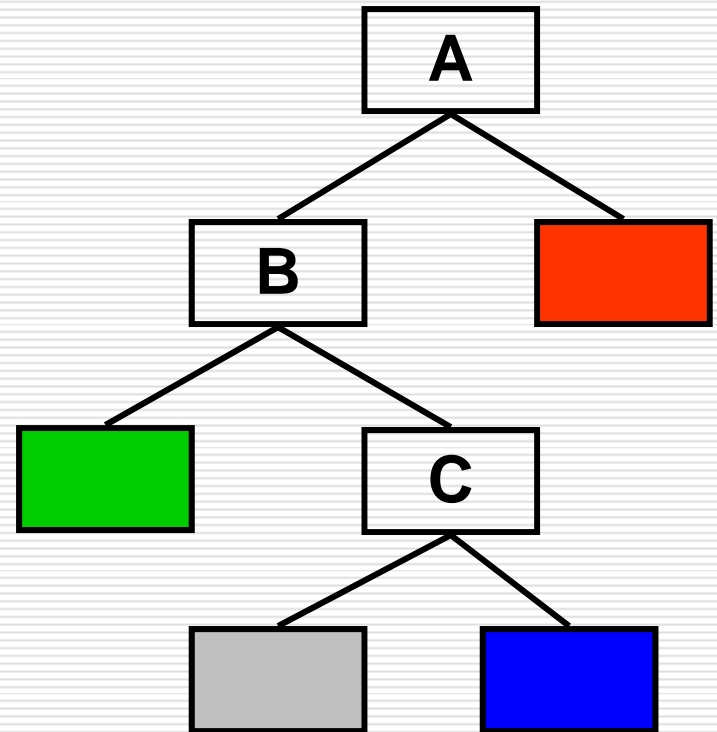
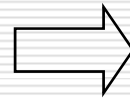
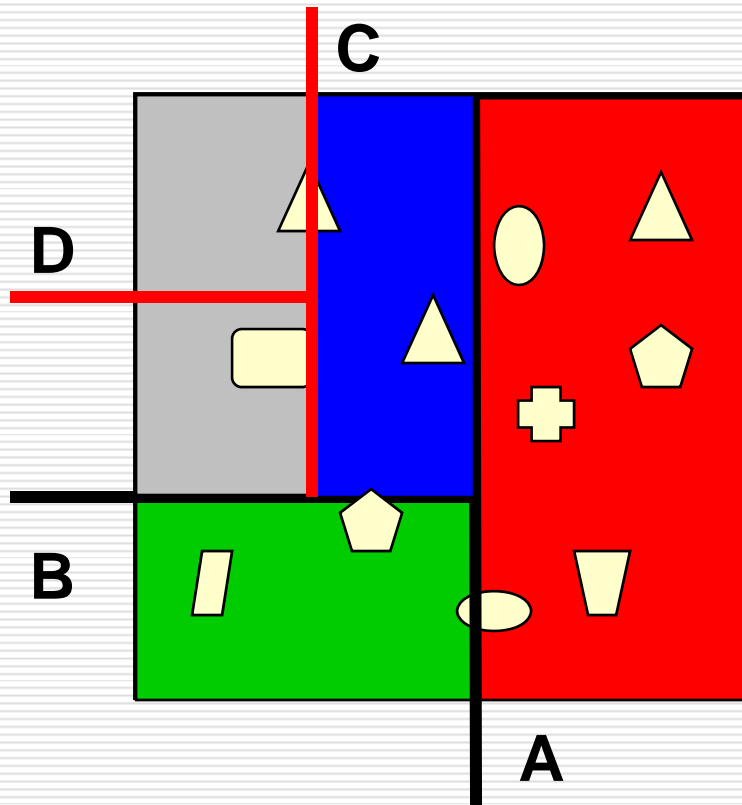
K-d Tree



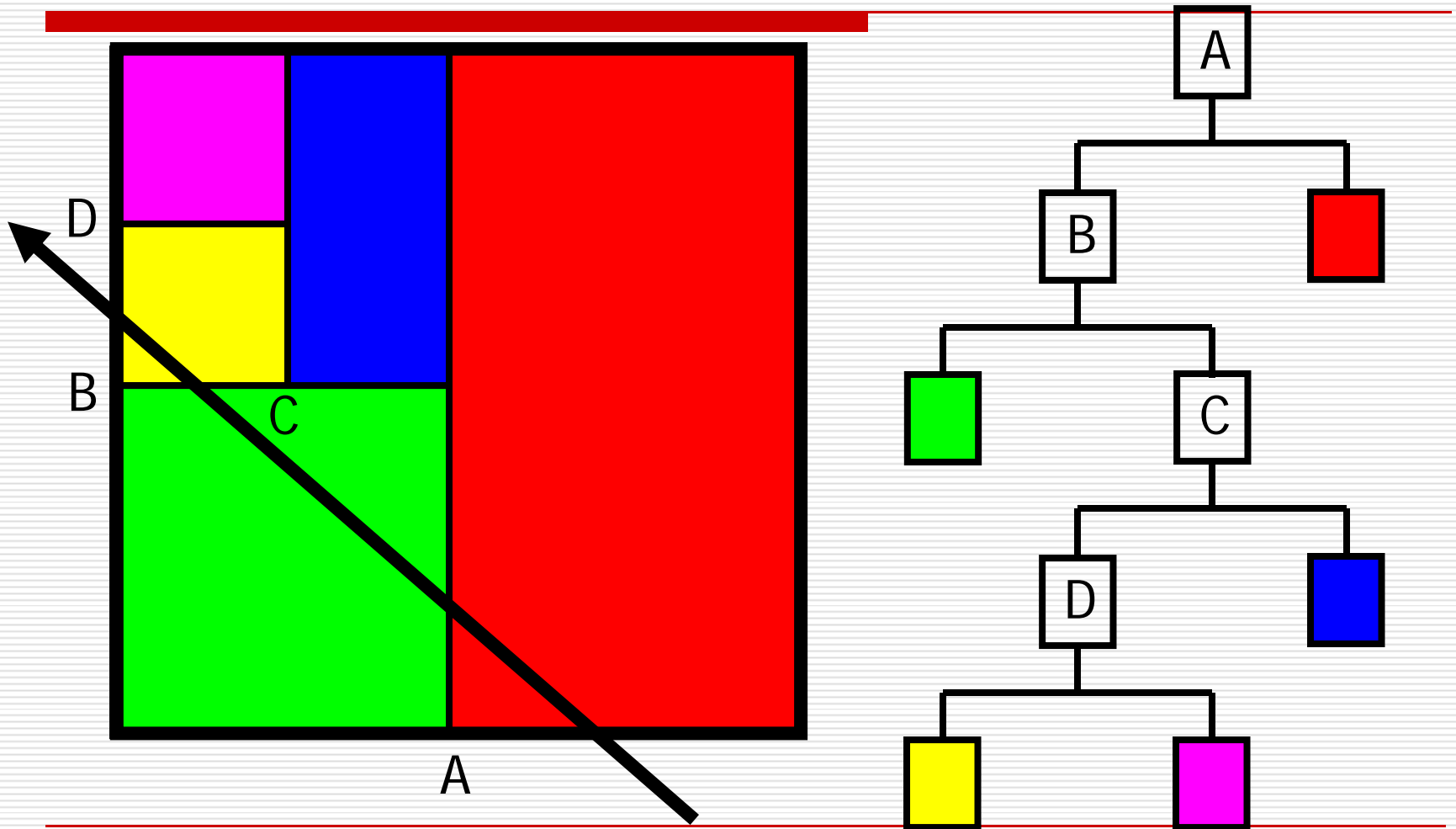
K-d Tree



K-d Tree

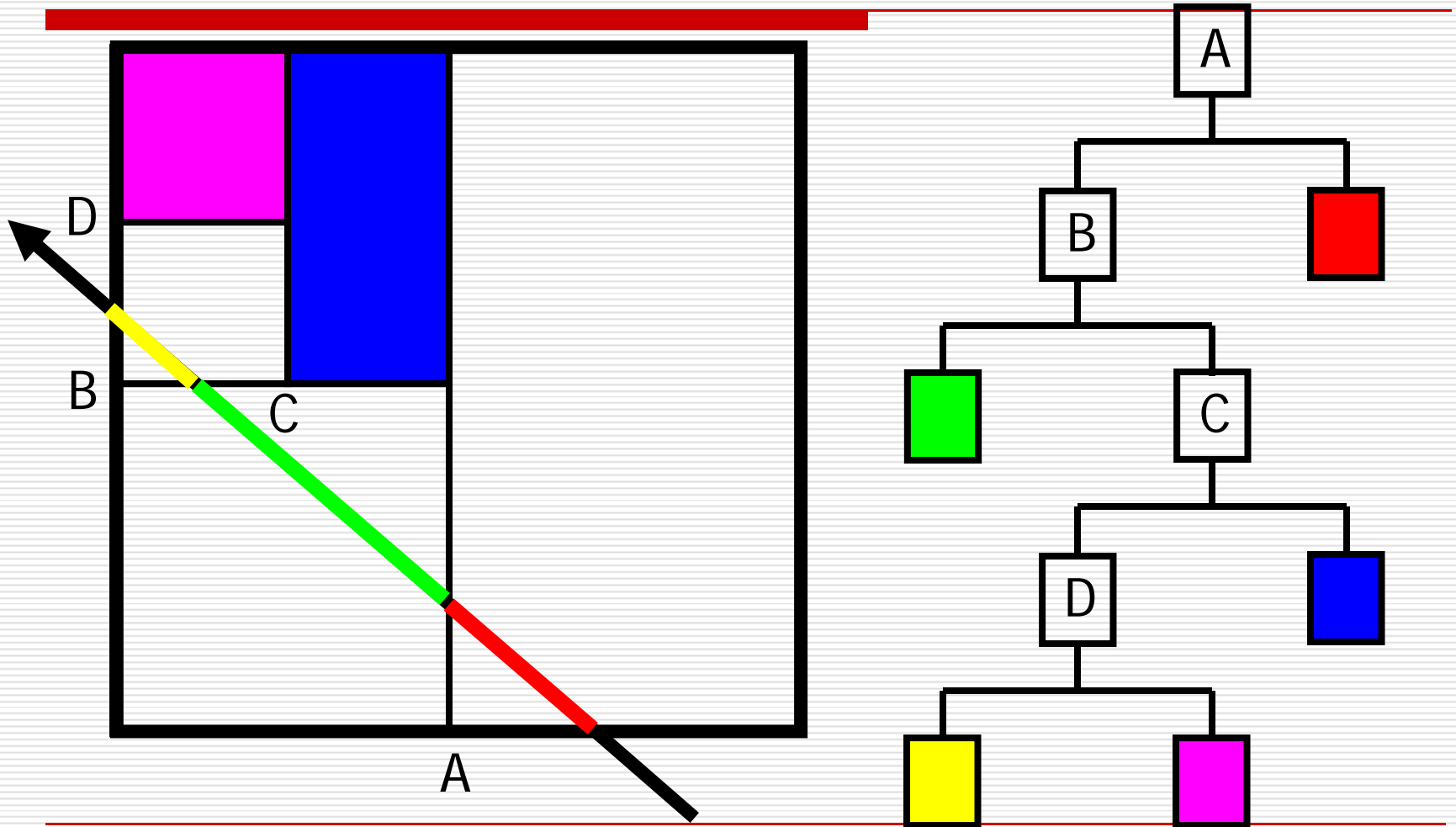


K-d Tree



Leaf nodes correspond to unique regions in space

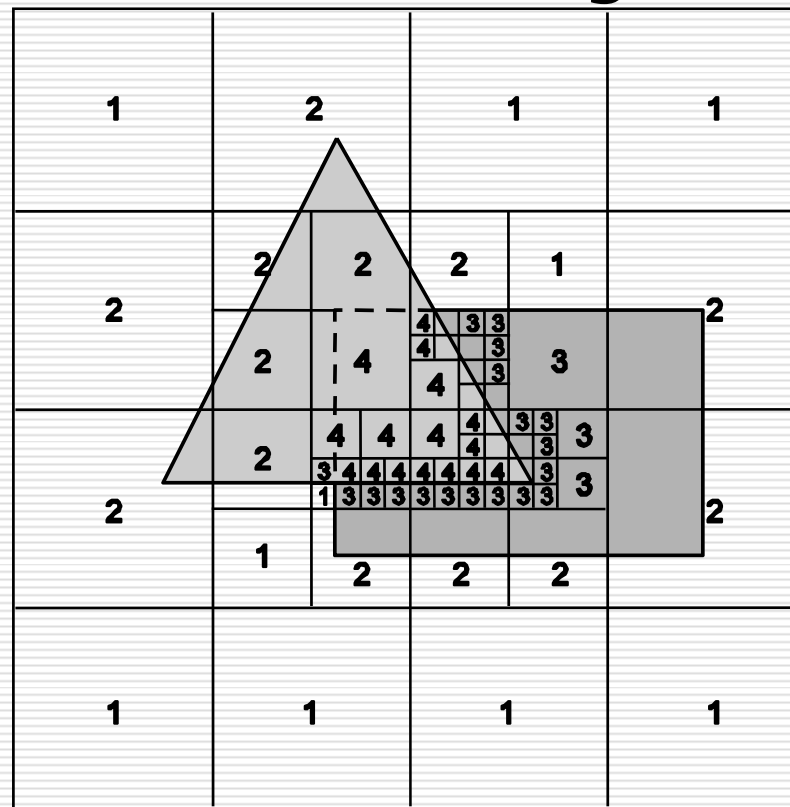
K-d Tree Traversal



Leaf nodes correspond to unique regions in space

Warnock's Algorithm

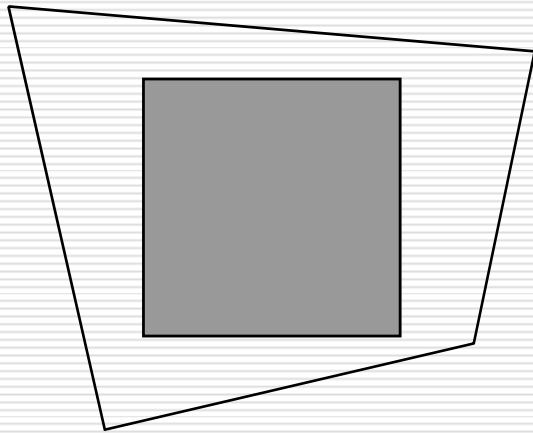
- an area-subdivision algorithm



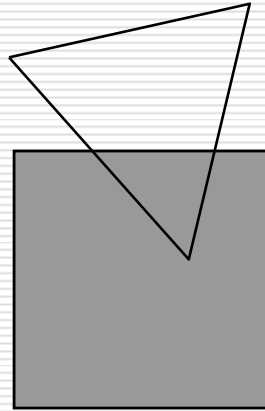
Warnock's Algorithm

1. all the polygons are disjoint from the area
 2. there is only one intersecting or only one contained polygon
 3. there is a single surrounding polygon, but no intersecting or contained polygons
 4. more than one polygon is intersecting, contained in, or surrounding the area, but one is a surrounding polygon that is in front of all the other polygons
-

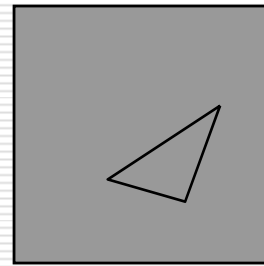
Warnock's Algorithm



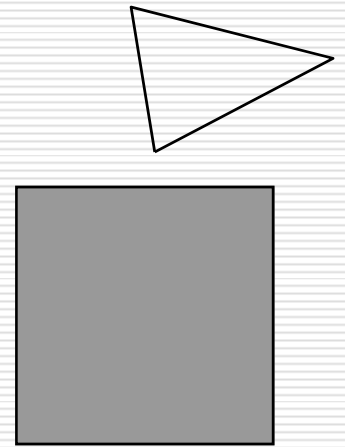
surrounding



intersecting

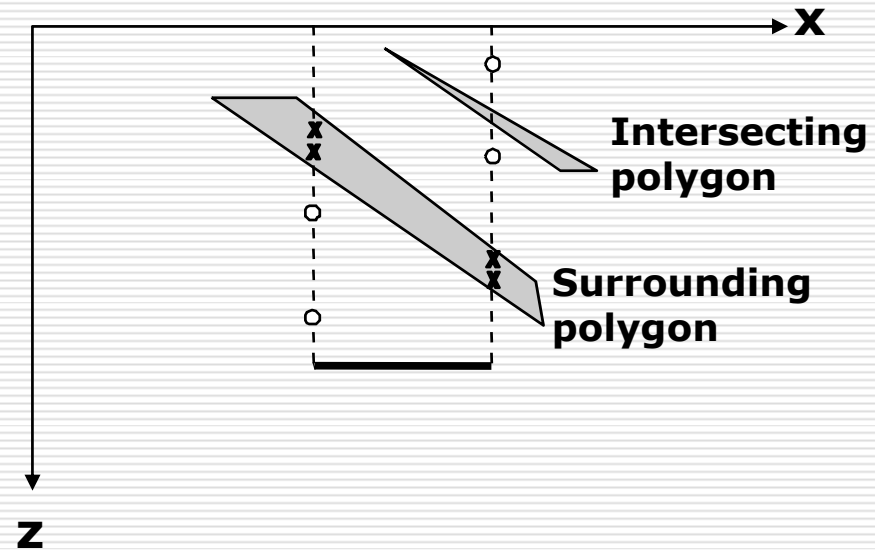
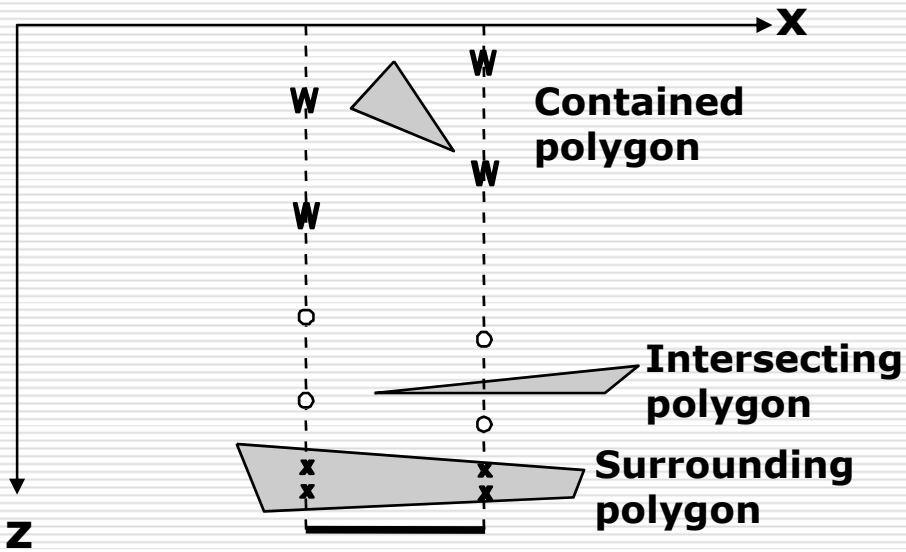


contained



disjoint

Warnock's Algorithm



Performance of Four Algorithms for Visible-Surface Determination

Algorithm	Number of Polygons		
	100	2,500	60,000
Depth sort	1	10	507
z-buffer	54	54	54
Scan line	5	21	100
Warnock area subdivision	11	64	307